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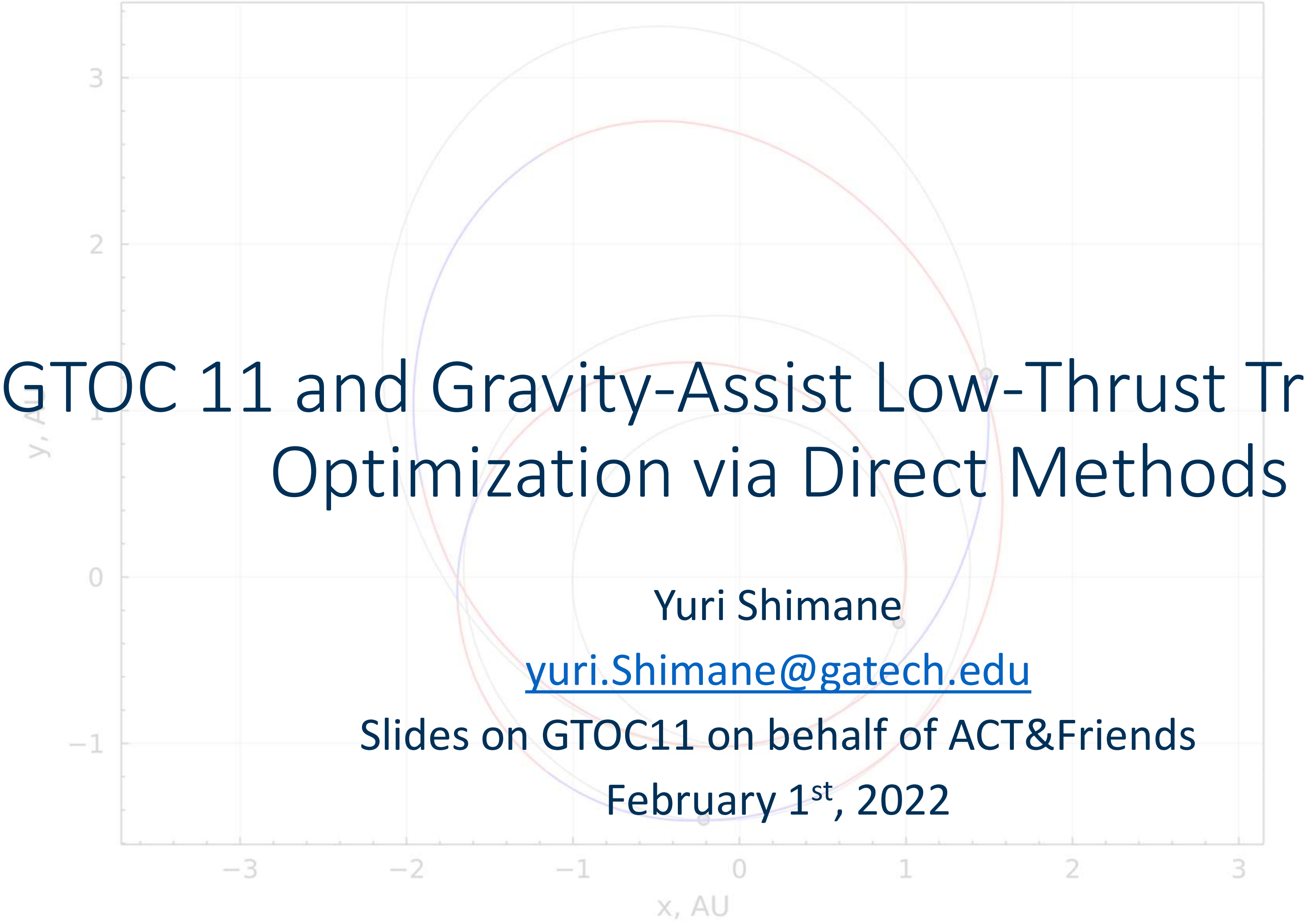

GTOC 11 and Gravity-Assist Low-Thrust Trajectory Optimization via Direct Methods

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Slides on GTOC11 on behalf of ACT&Friends

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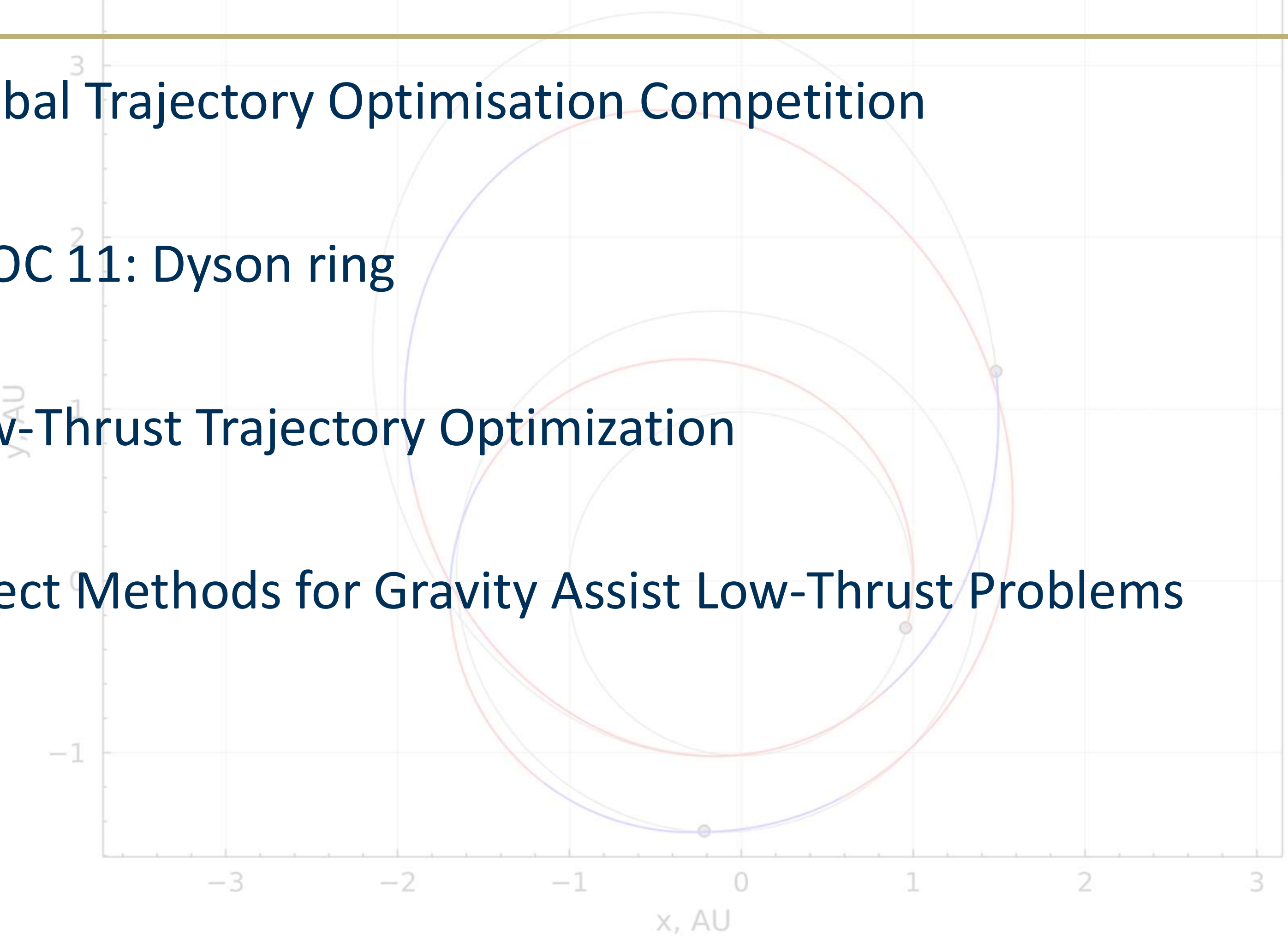
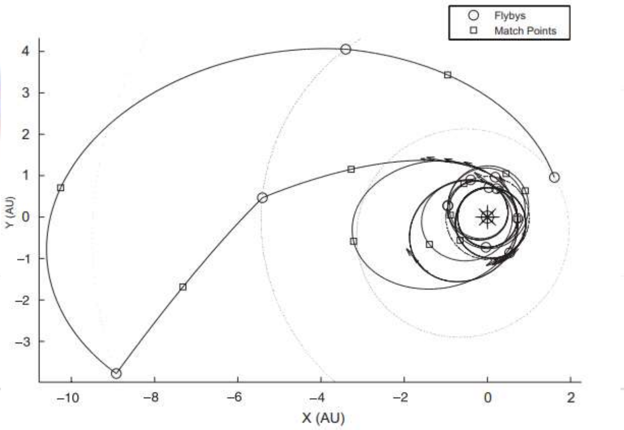
Presentation Outline



1. Global Trajectory Optimisation Competition
2. GTOC 11: Dyson ring
3. Low-Thrust Trajectory Optimization
4. Direct Methods for Gravity Assist Low-Thrust Problems

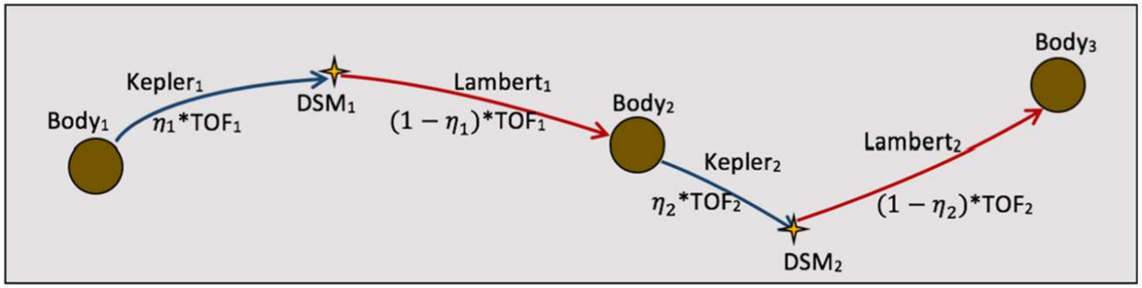
Global Trajectory Optimisation Competition

- Idea of GTOC:
  - <u>Well-defined</u> trajectory optimization problem
  - 1 month to solve “<u>exceptionally hard</u>” problem
  - Bi-yearly competition, hosted by previous edition winner
- Common characteristics:
  - High-thrust/low-thrust, gravity assists
  - Strong <u>combinatorial</u> flavor to problem



GTOC1 winning solution from JPL

Past editions and methods born

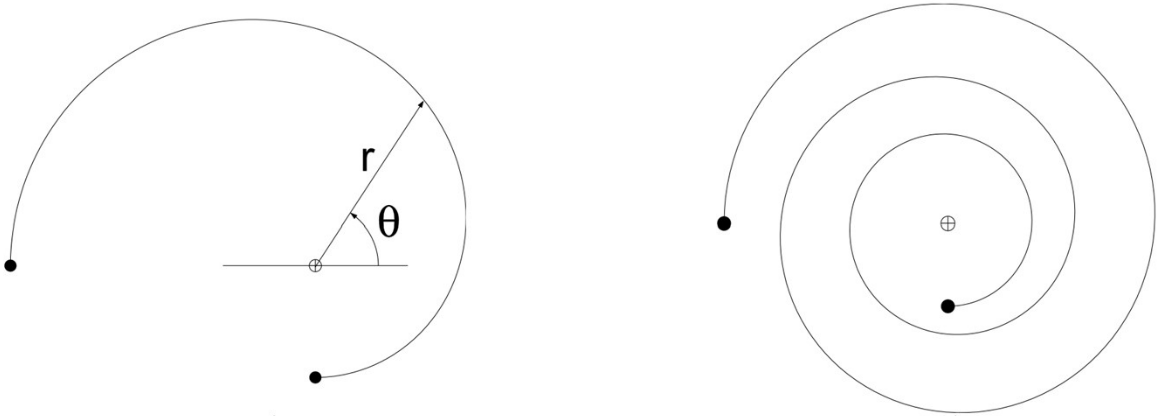


MGA-1DSM, Napier et al 2020

$$Q = (1 + W_p P)\sum_{\text{œ}} W_{\text{œ}} S_{\text{œ}} \left[\frac{d(\text{œ}, \text{œ}_T)}{\dot{\text{œ}}_{xx}}\right]^2$$

$$f^{\mathrm{LT}} = -\mathbf{\Psi}^T \left(\frac{\partial Q}{\partial \mathbf{X}}\right)^T$$

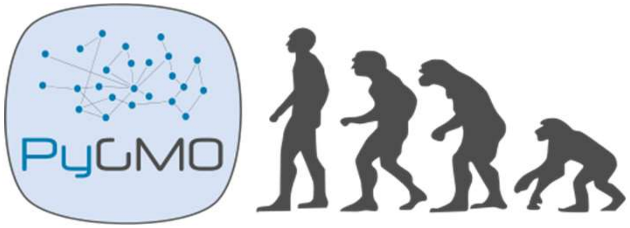
Q_Law, Petropoulos 2003, 2004



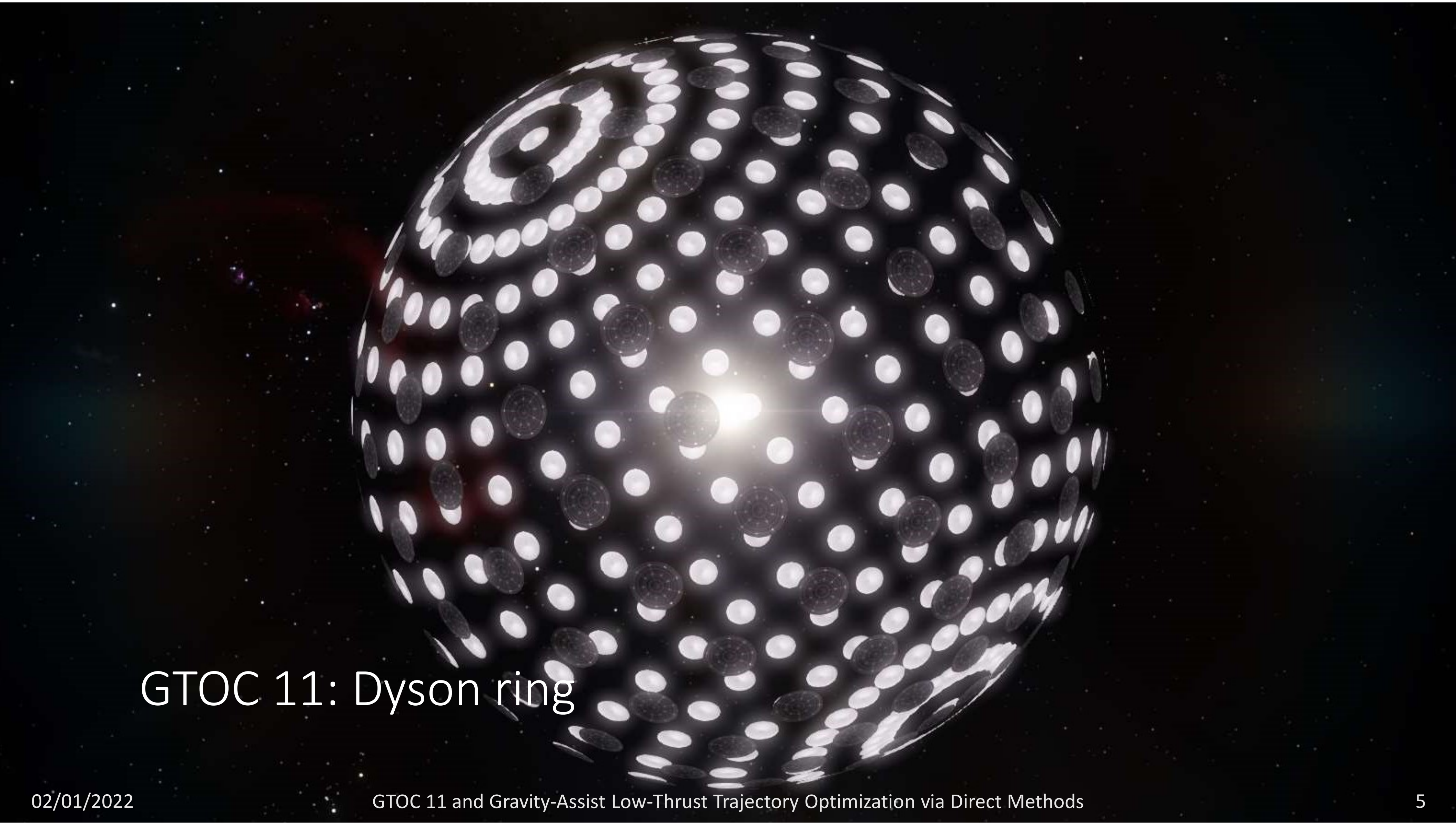
$k_2 = 2/3$

$k_2 = 2/11$

Shape-based methods, Petropoulos 2014



Packages: **pygmo**, **pykep**…



GTOC 11: Dyson ring

GTOC11: Constructing a Dyson ring

- Build **12 “stations”** equally spaced along a circular heliocentric orbit (orbit’s SMA, inclination, RAAN, and relative phasing are free variables) within a 20 years period
- Asteroids are used as building materials, and must be transferred to each station
- **10 “motherships”** are sent to visit as many among the **83453 asteroids**
- Once an asteroid is visited, it can be brought to a station by thrusting while consuming its own mass
- Stations must be built sequentially, with a 90 days window between the construction of each station
- Objective:

$$\max J = B\frac{10^{-10}M_{min}}{a_{Dyson}^{2}\sum_{k=1}^{10}\left(1+\Delta V_k^{Total}/50\right)^2}$$

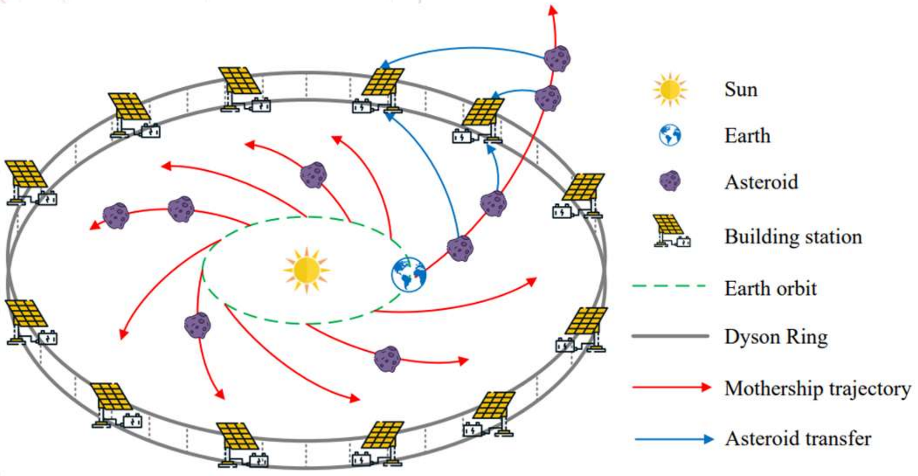
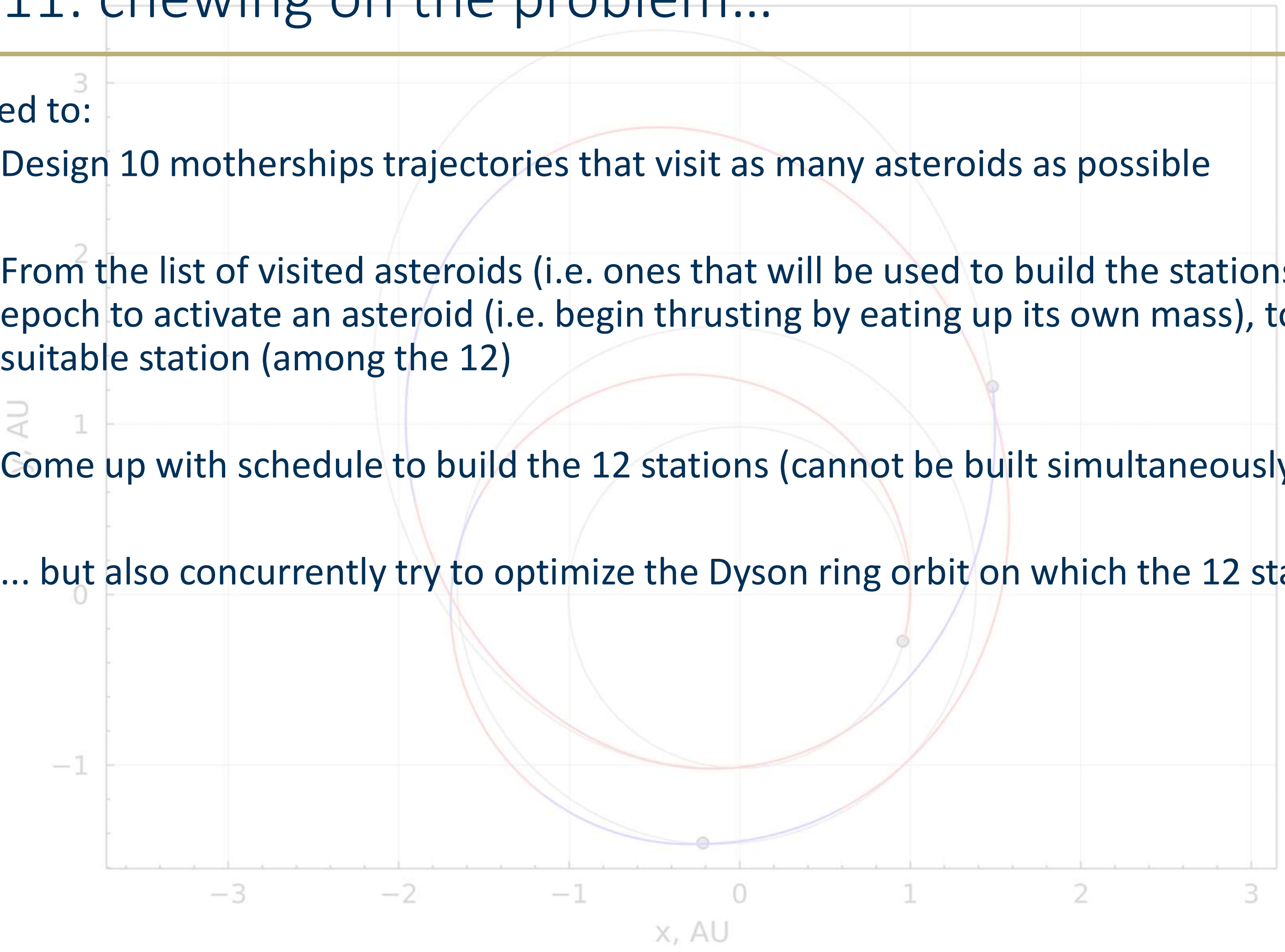


Figure 1 Illustration of the construction of the “Dyson ring”.

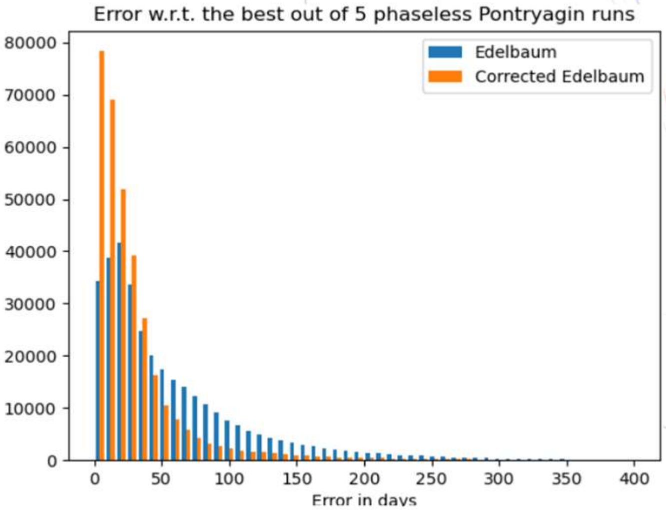
GTOC11: chewing on the problem…

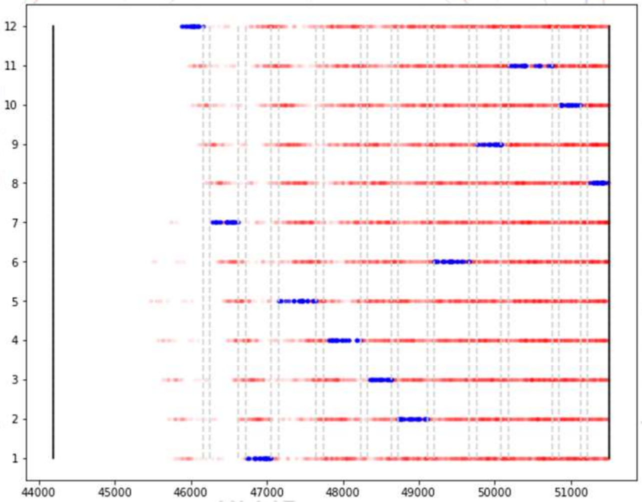
- We need to:
  1. Design 10 motherships trajectories that visit as many asteroids as possible
  2. From the list of visited asteroids (i.e. ones that will be used to build the stations), work out: (a) the best epoch to activate an asteroid (i.e. begin thrusting by eating up its own mass), to send to (b) the most suitable station (among the 12)
  3. Come up with schedule to build the 12 stations (cannot be built simultaneously!)
  4. ... but also concurrently try to optimize the Dyson ring orbit on which the 12 stations will be placed!

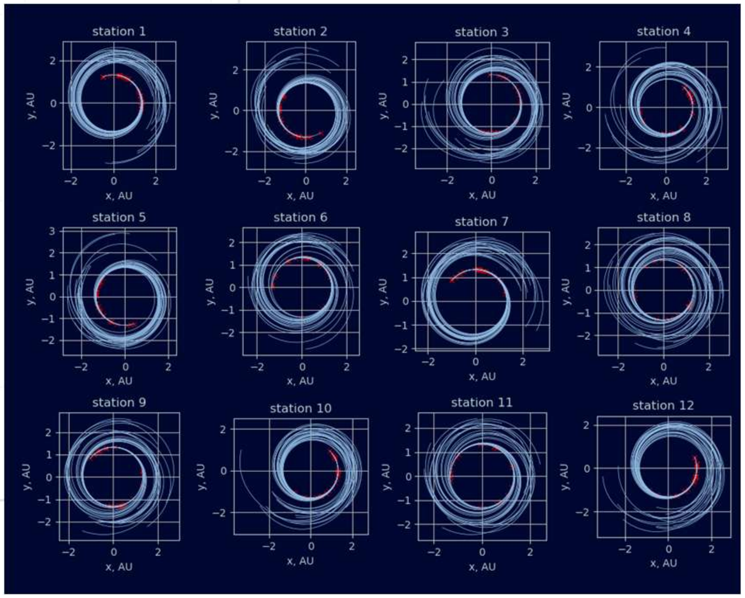


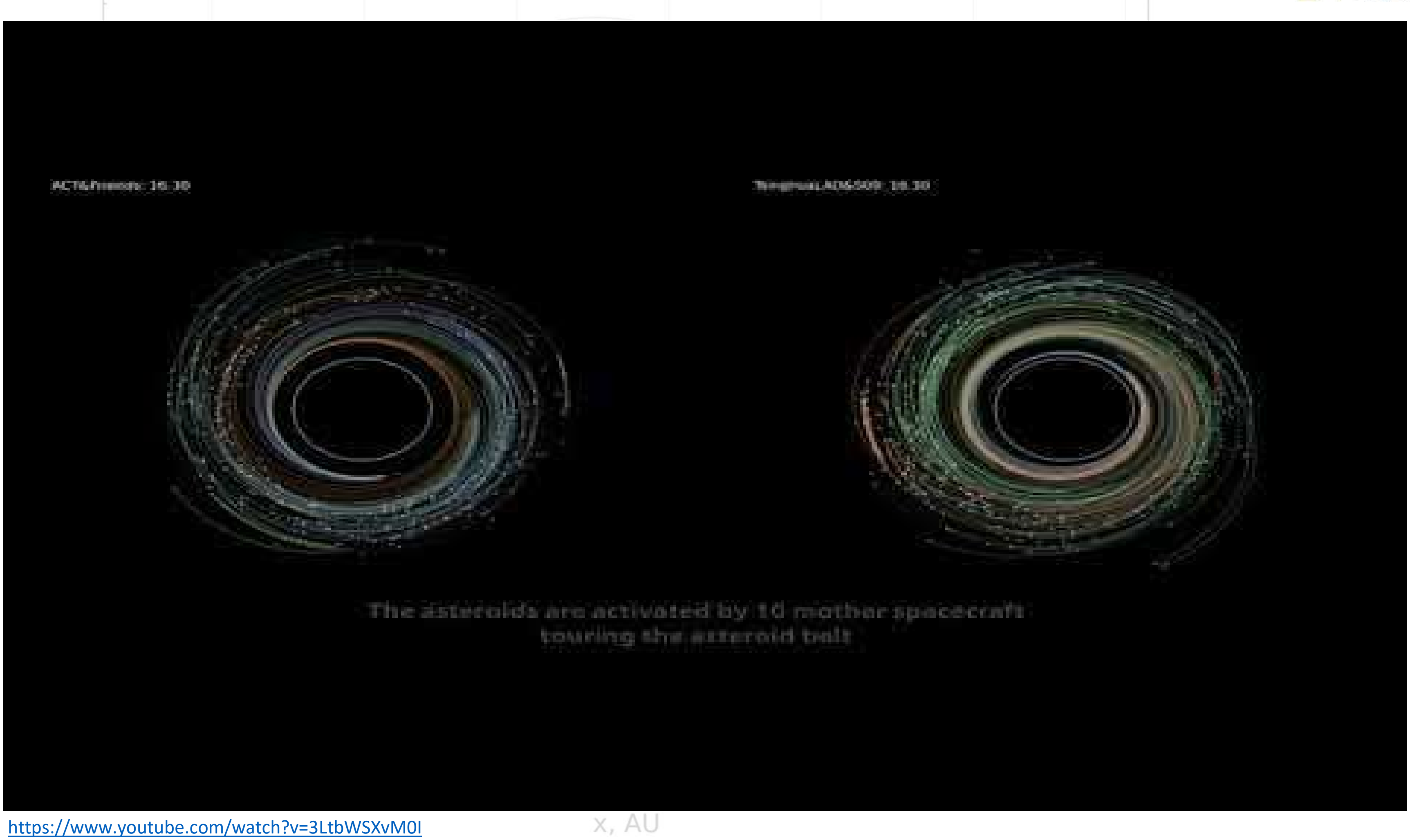
GTOC11: our strategy

- Three components to the problem:
  1. Motherships trajectory optimization (MGA-1DSM, P-ACO)
  2. Asteroids trajectory optimization to stations (low-thrust with no coasting)
     - Seeding initial guess with “corrected” Edelbaum approximation
     - Compute “phasing matrix”: transfers from all asteroids to all 12 stations, optimally phased (may be multiple of them!)
  3. Scheduling







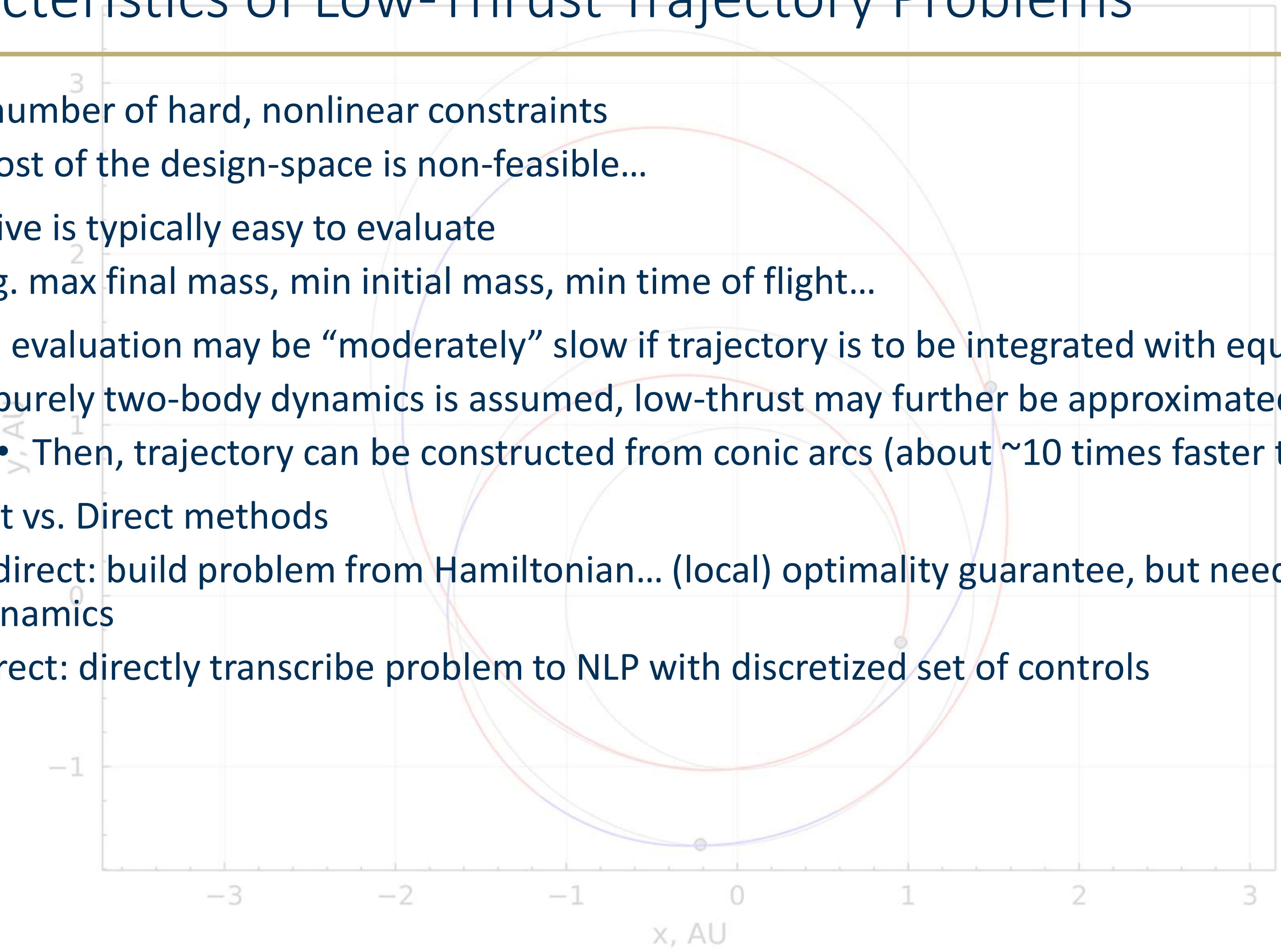


https://www.youtube.com/watch?v=3LtbWSXvM0I

Low-Thrust Trajectory Optimization

Credit: NASA/JPL

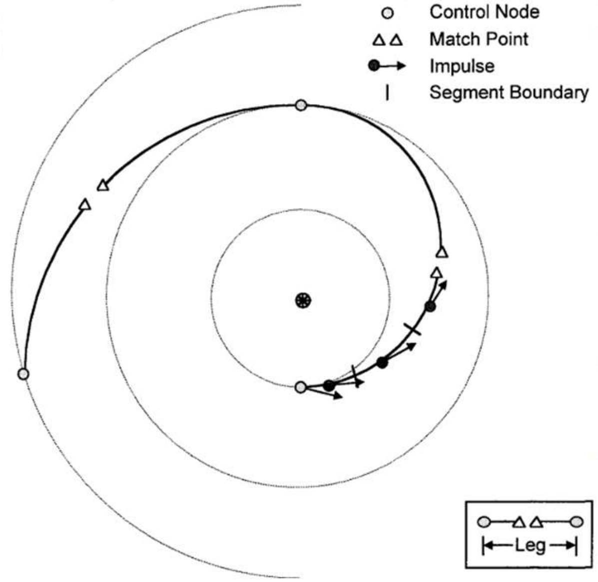
Characteristics of Low-Thrust Trajectory Problems



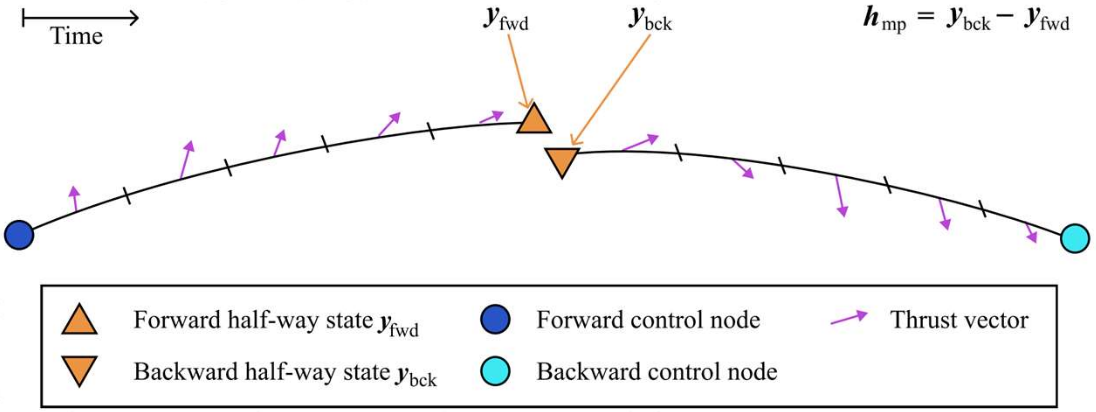
- Large number of hard, nonlinear constraints
  - Most of the design-space is non-feasible…
- Objective is typically easy to evaluate
  - E.g. max final mass, min initial mass, min time of flight…
- Fitness evaluation may be “moderately” slow if trajectory is to be integrated with equations of motion
  - If purely two-body dynamics is assumed, low-thrust may further be approximated as series of impulses
    - Then, trajectory can be constructed from conic arcs (about ~10 times faster than integration)
- Indirect vs. Direct methods
  - Indirect: build problem from Hamiltonian… (local) optimality guarantee, but need to handle co-state dynamics
  - Direct: directly transcribe problem to NLP with discretized set of controls

Sims-Flanagan Transcription: Overview

- Discretize trajectory into ***legs***, each beginning and ending at ***control nodes*** representing planet “visits”
  - First “visit” is the launch, last “visit” is the arrival, any intermediate “visits” are fly-by’s
  - State of spacecraft at each control-node is part of decision vector via epoch + v-infinity(ies)
- Each leg is discretized into ***segments***, where a control-law is defined for each segment
- Low-thrust may be approximated as *series of small impulses*, each located at the middle of each segment



From Sims and Flanagan, 1999



Sims-Flanagan Transcription: Problem

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & -m_f \\ \text{such that} \quad & \boldsymbol{h}_{\text{mp}} = 0 \\ & g_{\text{tof}} \leq 0 \\ & \boldsymbol{g}_{\text{fly-by}} \leq 0 \end{aligned}$$

$$\boldsymbol{x} = [\boldsymbol{c}_{\text{launch}},\, \boldsymbol{c}^{1}_{\text{fly-by}},\, \ldots,\, \boldsymbol{c}^{N-1}_{\text{fly-by}},\, \boldsymbol{c}_{\text{arrival}},\, \boldsymbol{\tau}^{1},\, \ldots,\, \boldsymbol{\tau}^{N}]$$

$$\boldsymbol{c}_{\text{launch}} = [t_0, m^1, v^1_\infty, \alpha^1, \delta^1]$$

$$\boldsymbol{c}^{i}_{\text{fly-by}} = [\Delta t^i, m^{i+1}, v^{i+1}_\infty, \alpha^{i+1}_-, \delta^{i+1}_-, \alpha^{i+1}_+, \delta^{i+1}_+]$$

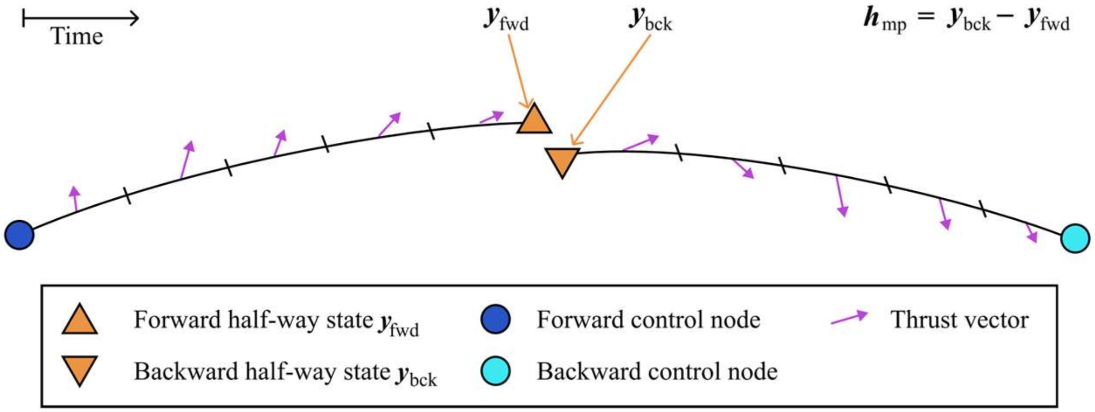
$$\boldsymbol{c}_{\text{arrival}} = [\Delta t^N, m^{N+1}, v^{N+1}_\infty, \alpha^{N+1}, \delta^{N+1}]$$

$$\boldsymbol{\tau}^j = [\tau^j_1,\, \theta^j_1,\, \beta^j_1,\, \ldots,\, \tau^j_k,\, \theta^j_k,\, \beta^j_k,\, \ldots,\, \tau^j_n,\, \theta^j_n,\, \beta^j_n]\,,\ \ j \in [1, N]\ , k \in [1, n]$$

$$\boldsymbol{h}^j_{mp} = \boldsymbol{y}_{\text{bck}} - \boldsymbol{y}_{\text{fwd}} = 0\,,\ \ j \in [1, N]$$

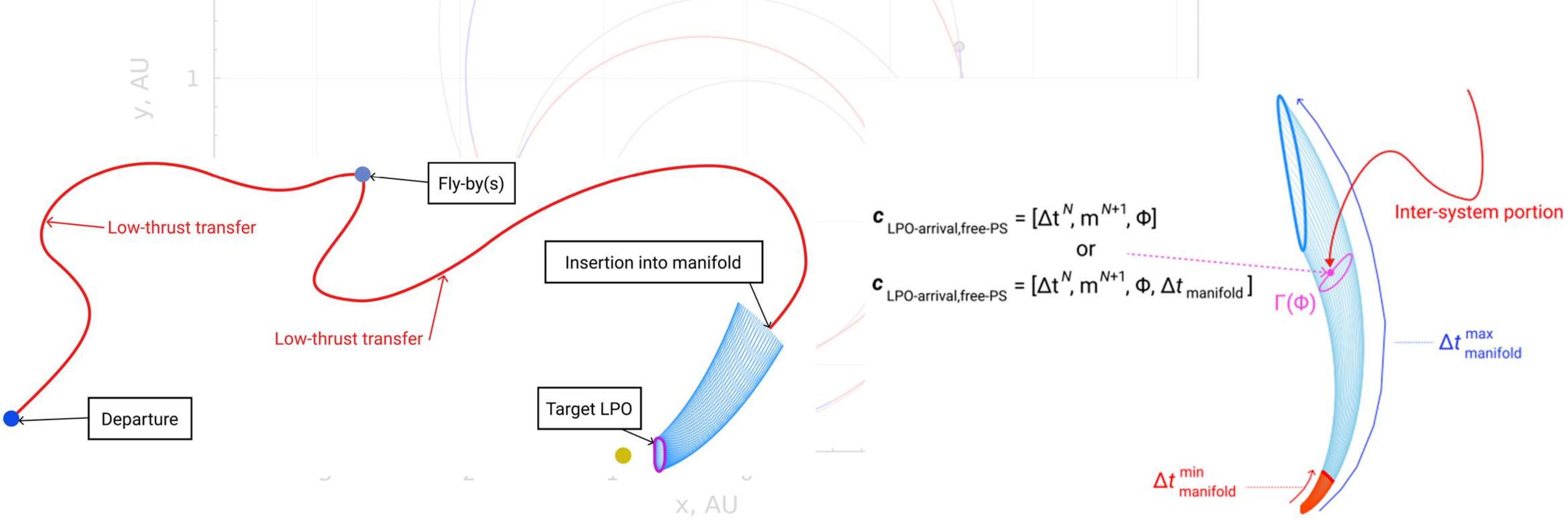
$$g_{\text{tof}} = \text{tof} - \text{tof}_{\max} = \sum_{j=1}^{N} \Delta t_j - \text{tof}_{\max} \leq 0$$

$$\boldsymbol{g}_{\text{fly-by}} = (r_{\text{planet}} + h_{\text{safe}}) - \frac{\mu_{\text{planet}}}{v^2_\infty}\left[1/\sin\left(\frac{\delta_{\text{turn-angle}}}{2}\right) - 1\right] \leq 0$$



Manifold Captures and Departure/Arrivals

- Replace arrival control node by insertion into manifold
- Ensure switch-point between two-body and three-body dynamics occurs “sufficiently far” such that discrepancy is negligible
- Advantage using manifold: no insertion braking required -> applicable for low-thrust only spacecraft



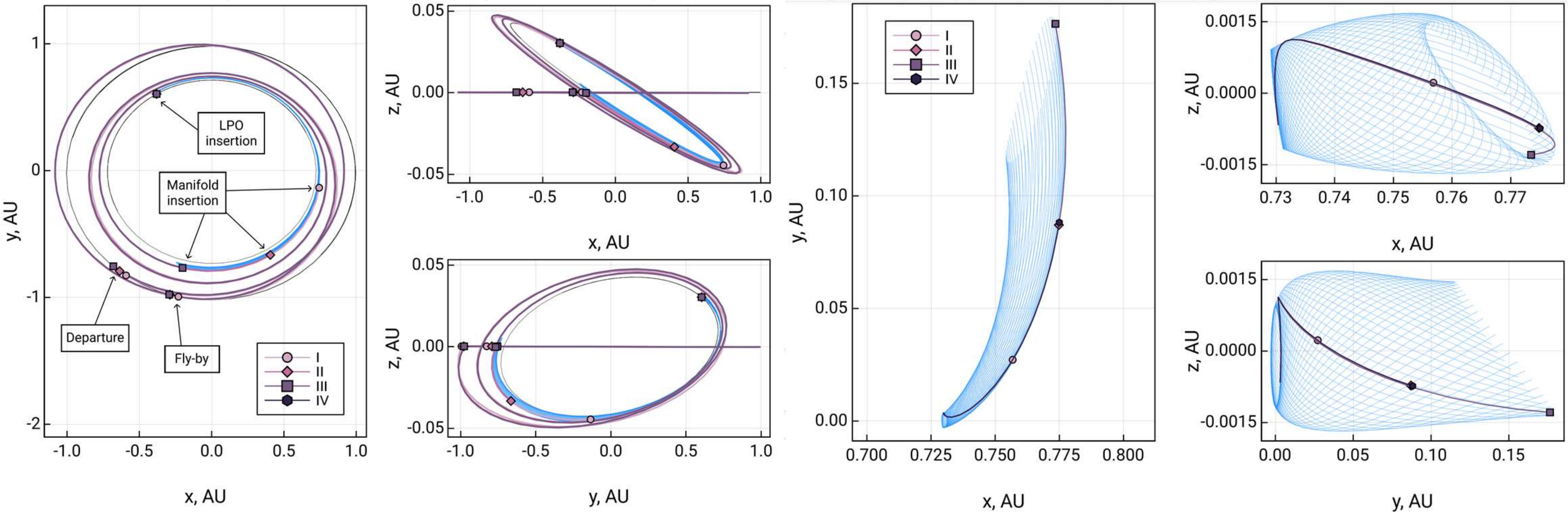
$$\mathbf{c}_{\text{LPO-arrival,free-PS}} = [\Delta t^{N}, m^{N+1}, \Phi]$$

or

$$\mathbf{c}_{\text{LPO-arrival,free-PS}} = [\Delta t^{N}, m^{N+1}, \Phi, \Delta t_{manifold}]$$

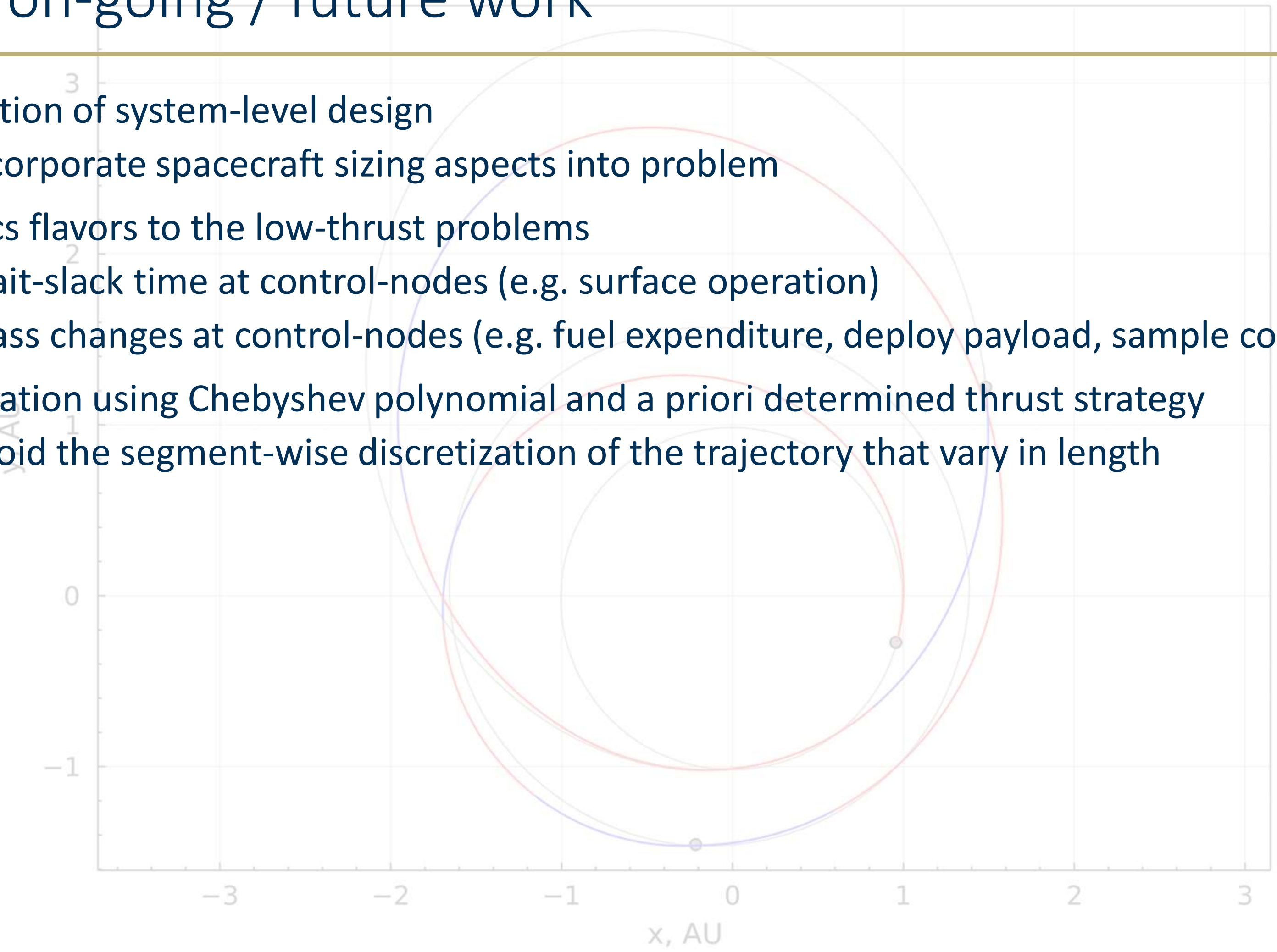
Case Study: Sun-Venus L2 halo with EGA

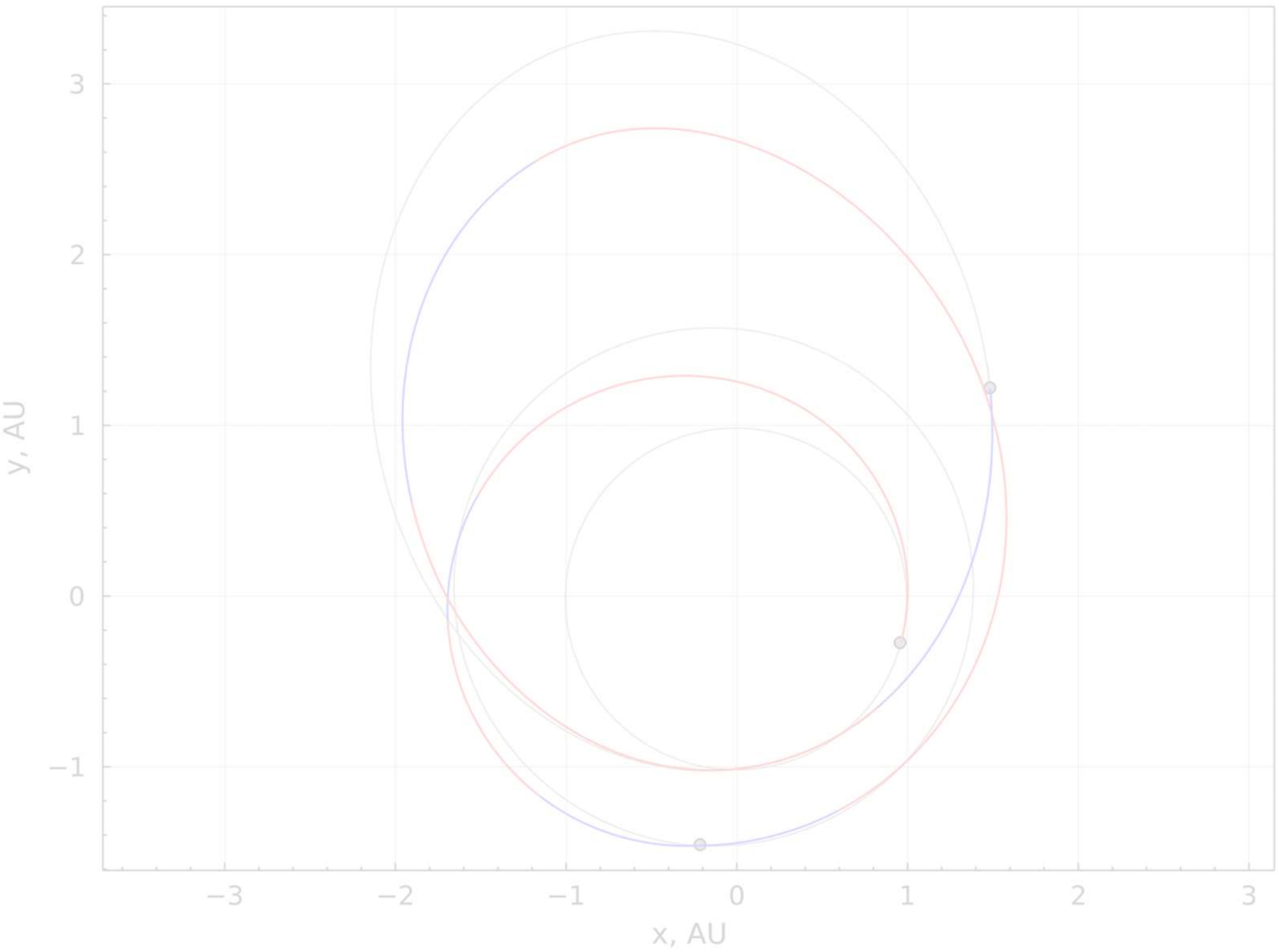
- Earth – Earth – Sun-Venus L2 halo orbit
  - Launch window 5/1/2022 – 5/1/2024, v-infinity 1.5 km/s
  - 4100 kg wet mass, Isp 3500 s, max thrust 0.4 M



Some on-going / future work

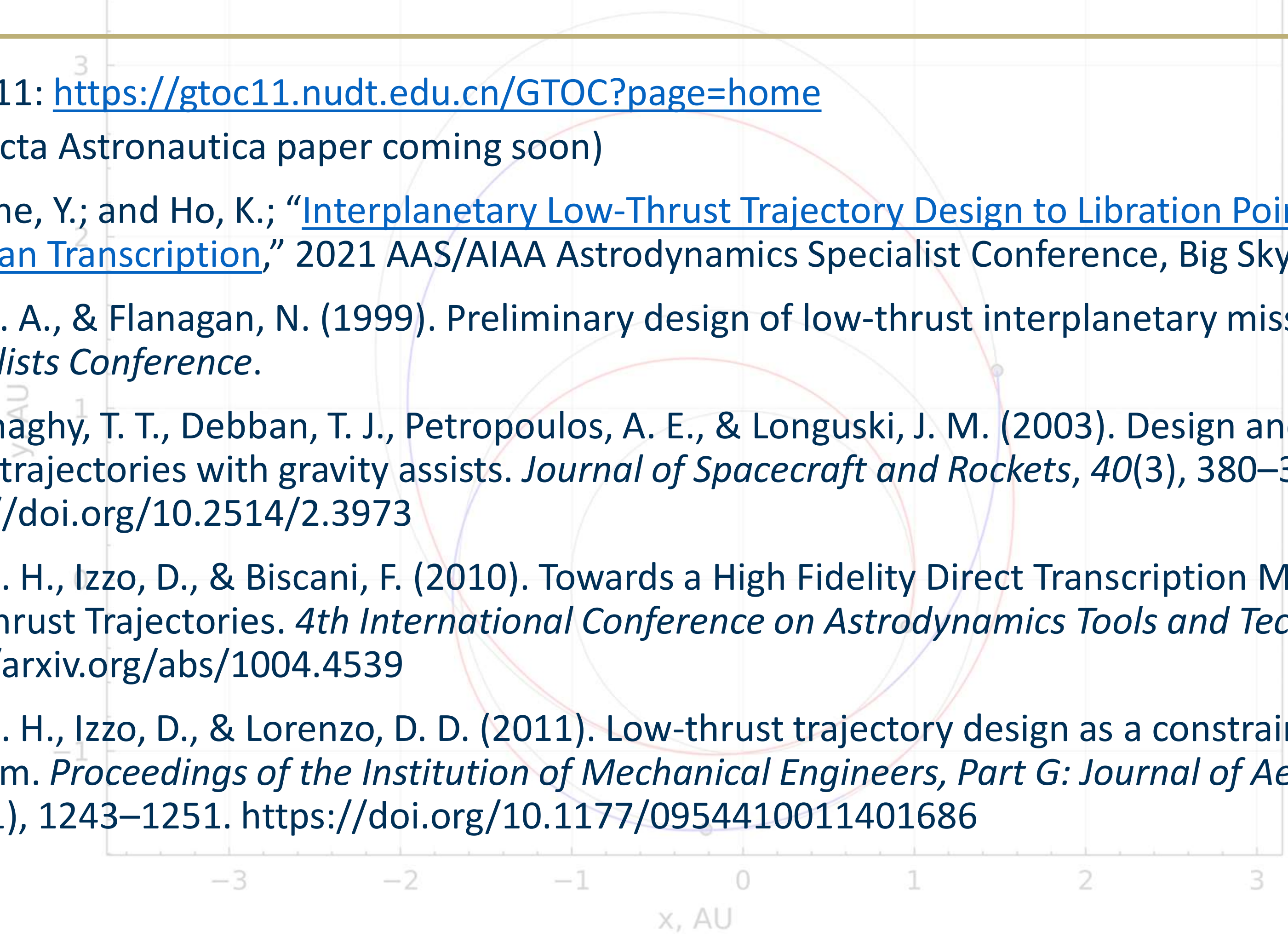
- Integration of system-level design
  - Incorporate spacecraft sizing aspects into problem
- Logistics flavors to the low-thrust problems
  - Wait-slack time at control-nodes (e.g. surface operation)
  - Mass changes at control-nodes (e.g. fuel expenditure, deploy payload, sample collection, refueling…)
- Formulation using Chebyshev polynomial and a priori determined thrust strategy
  - Avoid the segment-wise discretization of the trajectory that vary in length





Thank you!

Some References



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