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SURVEY OF GLOBAL OPTIMIZATION METHODS FOR LOW-THRUST, MULTIPLE ASTEROID TOUR MISSIONS

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Electric propulsion has recently become a viable option for robotic missions, enabling shorter flight times, fewer required planetary gravity assists, smaller launch vehicles, and/or larger payloads. Trajectory design of these missions often relies on local optimization of the low-thrust trajectories using starting points for departure and arrival dates and selection of gravitational swing-bys based on previous experience. Global optimization of a low-thrust trajectory with multiple targets and gravity assists, however, is a difficult problem, due to the multi-modality and large size of the design space. In choosing analysis techniques, there exists an important tradeoff between the accuracy of the results and computing time required. This paper presents the difficulty of solving this global optimization problem, using the design of a multiple asteroid tour mission as an example. Furthermore, this paper presents an overview of the methods available for both low-thrust trajectory optimization and global optimization, along with recent improvements made, and assesses their efficacy and applicability to solving a multiple target/multiple gravity assist problem.

INTRODUCTION

With the recent successful launches of the Deep Space 1[1], SMART-1[2], and Hayabusa[3] missions, and with the upcoming launch of Dawn[4], electric propulsion has become a viable option for robotic solar system exploration. Previous studies have shown that the use of electric propulsion has the potential to result in shorter flight times, fewer required planetary gravity assists, and/or smaller launch vehicles[5]. One major challenge of low-thrust missions is in the area of trajectory design and optimization. Trajectory design often relies on local optimization of the low-thrust trajectories using expert-based starting points for departure and arrival dates and selection of gravitational swing-bys. These choices are generally based on known configurations that have worked well in previous analyses or simply on trial and error. Global optimization, however, is a more difficult problem, especially when multiple target bodies and gravitational assists are added. In choosing an analysis technique, there exists an important tradeoff between the accuracy of the results and computing time required. Over the past several years, numerous improvements have been made in the area of both low-thrust trajectory optimization and the application of global optimization methods to the low-thrust problem.

Of particular interest is the application of these optimization methods to a multiple asteroid tour mission. With the exception of SMART-1, all of the aforementioned missions either intercepted or flew-by asteroids and/or comets. There is significant interest in studying Near-Earth Asteroids (NEAs) in particular, because of the possibility of an Earth impact and because of their connection to the formation of the solar system and life on Earth. The NEAR mission, for example, which orbited the asteroid 433 Eros,

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was interested in answering questions related to the nature and origin of near Earth objects, for several reasons[6]. First, they are the primary source of large body collisions with Earth, thereby influencing evolution of the atmosphere and life. Second, asteroids provide clues to the nature of the early solar system processes and conditions, as these are often preserved on small bodies such as asteroids, comets, or meteorites. The NEAs are especially believed to contain clues to the nature of the building blocks from which the inner planets were formed. Finally, the NEAR mission was interested in measuring the properties of 433 Eros, in order to establish a connection between meteorites and the history of asteroids, to better quantify the nature of their impact hazard to Earth.

To date, over four thousand NEAs have been identified, 841 of which have been identified as Potentially Hazardous Asteroids (PHA)[7]. Figure 1 plots all of the NEAs, as a function of their semi-major axis and eccentricity, with the PHAs marked in red. An asteroid is defined as potentially hazardous if the minimum distance between its orbit and the Earth's orbit is less than 0.05 AU, and if its absolute magnitude (H), which is used to estimate the diameter of an asteroid, is less than 22. This value of H corresponds to a diameter of 110m – 240m or greater. Furthermore, missions to multiple asteroids are of particular interest in order to maximize the science return for a single spacecraft, as opposed to visiting just a single asteroid. Additionally, designing a mission to rendezvous with or flyby multiple asteroids is a more challenging trajectory optimization problem, which is why it has been chosen as the topic of this paper.

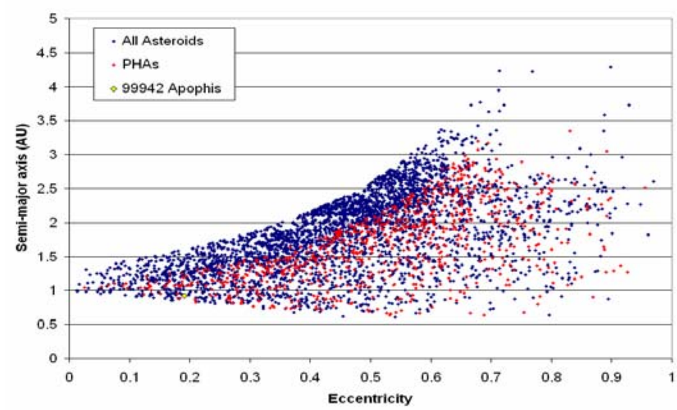

Figure 1 Plot of currently known near Earth asteroids.

PROBLEM OVERVIEW

The problem of interest is to determine the global optimum of a low-thrust, multiple asteroid tour mission. This will involve searching over a wide range of parameters, including Earth departure date, selection of the appropriate combination of asteroids, arrival dates at each of the asteroids, and so on. Additionally, local optimization will be required, in order to determine the thrust direction and magnitude along the spacecraft trajectory for a given set of global parameters. Figure 2 (left) plots the optimal low-thrust trajectory from Earth to the asteroid Apophis, departing Earth on 1 December 2012 with a 210-day time of flight. In this paper, optimal will be understood to mean minimum-fuel. Keep in mind that other optimality conditions could be chosen, such as minimum time of flight, but in general, the trajectory optimization problem is formulated as minimizing propellant consumption possibly with some constraint on maximum time of flight. While the trajectory shown in Figure 2 may be optimal for that particular launch date and time of flight, that does not guarantee a globally optimal solution over a range of departure dates and flight times. In fact, the final mass at Apophis was calculated to be 925kg for this particular trajectory, based on an initial mass of 1500kg. The global optimum over a two-year span from 2012-2013, with a maximum time of flight of 210 days, was found to be 990 kg. If a poor guess for departure date was chosen, however, also for the 210-day time of flight, the resulting arrival mass was as low as 54kg. This is due to improper phasing between Earth and Apophis for that particular departure date and time of flight.

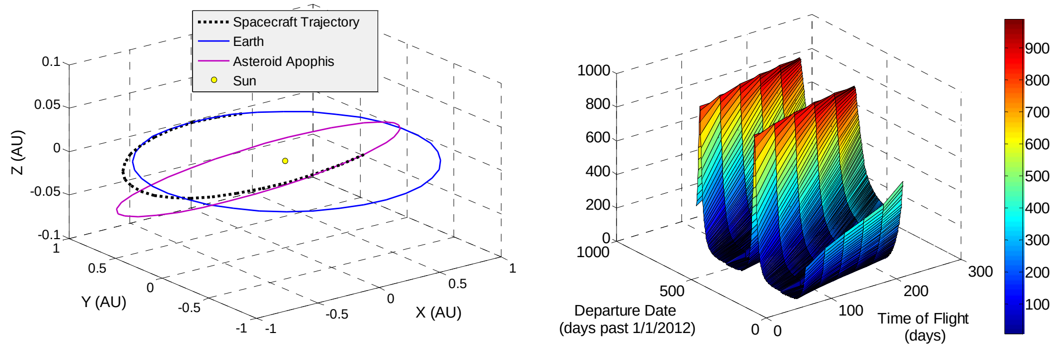

Figure 2 Trajectory to the asteroid Apophis (left) departing Earth on 1 December 2012 with a 210-day time of flight; arrival spacecraft mass (right) for a range of departure dates and times of flight, based on an initial mass of 1500 kg.

Figure 2 (right) plots the fuel-optimal trajectories for a sweep of departure dates and times of flight. Both the single trajectory and the surface plot results were generated using Chebytop, a low-fidelity program that uses Chebychev polynomials to represent state variables. The polynomials are then differentiated and integrated in closed form to solve a variable-thrust trajectory, which can be used to approximate the constant thrust trajectory[8]. As can be seen from the plot, the final mass at Apophis arrival is multi-modal with respect to departure date. Therefore, if a gradient-based optimizer were used to determine the Earth departure date along with the optimal control parameters, only a local optimum would be found, close to the initial guess given for departure date. If a wider range of departure dates were of interest – more than a year span for this particular trajectory, due to the phasing – either a similar sweep of departure dates would be required or an optimizer capable of handling multi-modal design spaces could be utilized. For such a simple problem, doing a parametric study over a single variable is a simple process, and not particularly time intensive, particularly with the low-fidelity and fast-running approach used in Chebytop. When more variables are added, however, the problem quickly increases in size and complexity.

As aforementioned, the full global optimization problem of a multiple asteroid tour mission includes many more free variables. Globally, the following are free variables: Earth departure date, asteroid 1 (chosen from some set of NEAs of interest), time of flight to asteroid 1, asteroid 1 stay time,…, asteroid *n*, time of flight to asteroid n, asteroid *n* stay time, and Earth arrival date (if desired). For a specific set of global variables, the optimal control history for the spacecraft must also be determined. The constraints chosen at each asteroid arrival will depend on whether a flyby or rendezvous is desired. For a flyby, of course, the stay time will be predetermined at a value of zero. Figure 3 illustrates a schematic of the asteroid tour problem for a set of three predetermined asteroids, including an Earth return segment. Only the projection onto the X-Y plane is shown.

With the increased number of design variables, the increasing dimensionality of the problem causes a parametric study to become prohibitive due to computational requirements. The parametric study plotted in Figure 2 discretized the departure dates in two-day increments and the time of flight in 30-day increments, resulting in a total of 2160 cases, where each of those cases includes a local trajectory optimization for the optimal control history. Table 1 illustrates a sample discretization for a subset of the design variables included in the full global optimization problem, where a sequence of three asteroids is predetermined and an Earth return is not included. Even for this subset, with a fairly course discretization, a grid search would require almost 30 million cases! If the choice and sequence of asteroids were not predetermined, but instead included in the optimization problem, this number would grow even larger. Additionally, gravity assists could also be included in the trajectory in an attempt to further reduce the required propellant mass. Keep in mind again that each of these cases requires an optimization of the spacecraft controls, which can be time intensive in and of itself. Therefore, a domain-spanning global optimization method should be chosen to avoid running all possible combinations of the discretized global variables.

Table 1
SAMPLE GRID SEARCH DISCRETIZATION FOR MULTIPLE ASTEROID TOUR PROBLEM.

Variable	Range	Increment	# Grid Point
Earth departure date	2 years	15 days	50
TOF to Asteroid 1	200-400 days	10 days	21
Stay time at Asteroid 1	30-60 days	10 days	4
TOF to Asteroid 2	200-400 days	10 days	21
Stay time at Asteroid 2	30-60 days	10 days	4
TOF to Asteroid 3	200-400 days	10 days	21
Stay time at Asteroid 2	30-60 days	10 days	4
TOTAL # OF CASES:			29,635,200

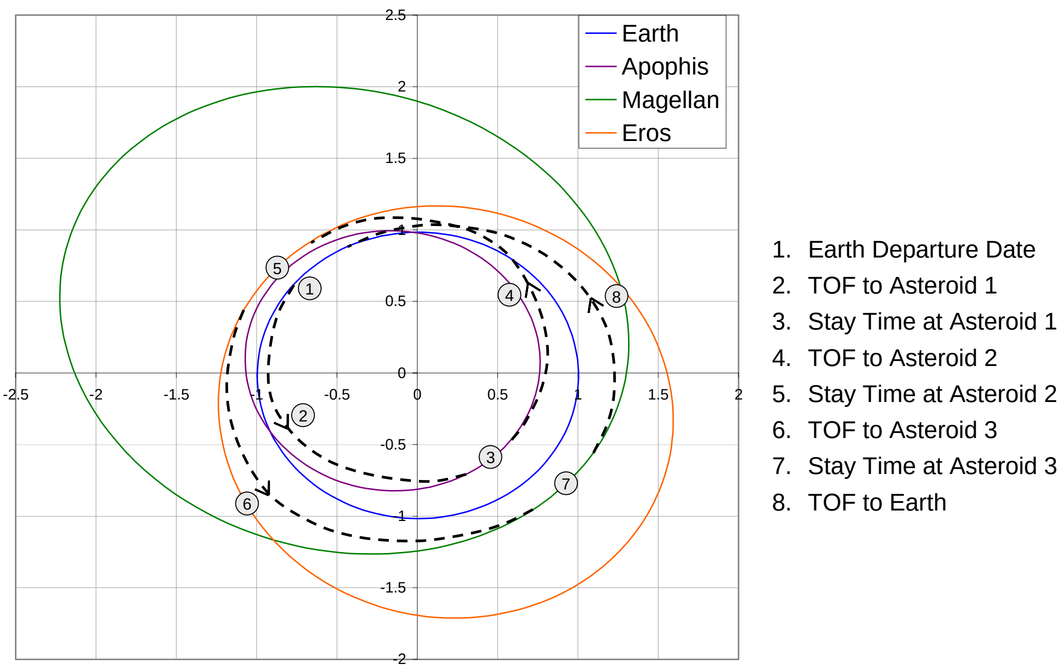

Figure 3 Illustration of asteroid tour optimization problem, for a predetermined asteroid sequence.

The multiple asteroid tour mission outlined above is analogous to multiple gravity assist trajectories to the outer planets, on which much work has already been done. In general, however, the trajectory optimization is based on a predetermined sequence of gravitational assist bodies, which would be comparable to an asteroid tour trajectory optimization with a predetermined sequence of asteroids. Furthermore, the solutions are often based on trial and error or on previously known configurations that yield a feasible solution, since an exploration of the full design space has been computationally prohibitive up to this point. Several sequences of gravitational assists are examined, with a range of departure dates and times of flight, until a good solution is found. This of course does not guarantee that the globally optimal solution has been found, and significant improvement in the objective function could be realized if an automated global optimization method were successfully applied to this problem. It is also important to note that while a planar assumption can often be made when considering trajectories to the outer planets (with the exception of Pluto), particularly during the conceptual design phase, many NEAs are highly inclined and this assumption is not valid. In fact, about 92% of all NEAs have inclinations greater than any of the outer planets, again with the exception of Pluto, and more than 50% have inclinations greater than 10°.

Overview of Desired Optimization Characteristics

Exploring a large design space is most important during the conceptual design phase of mission planning, which is also when accuracy is the least important. In order to quickly examine a large number of potential trajectories, a low-fidelity approach may be appropriate. The accuracy must be sufficient enough so that the location of the optimal solution is close to the actual optimal when a more accurate method is employed. However, errors of a few percent are acceptable at this phase. The purpose of conceptual design is to determine candidate trajectories that will best meet the mission constraints and objectives. More accurate trajectory optimization techniques can be used later in the design process on a handful of promising trajectories.

Because of the nature of the problem, two optimization methods will likely be required. The global optimization method must be domain spanning, in order to account for the multi-modality of the problem, and must be able to handle discrete variables. This global optimization method will handle the variables referred to as "global variables" above (Earth departure date, sequence of asteroids, times of flight, etc.). Another optimization method will have to be employed to optimize the physical trajectory, which includes the control parameters of the spacecraft (thrust magnitude and direction, and possibly power). This paper will refer to this optimization as local optimization. While many trajectory optimization programs include both optimization of the thrust and certain global parameters such as time of flight or departure date, this will result in only a locally optimal solution based on the initial guess for the global variables. For this reason, the global and local optimization methods will be differentiated in this paper.

TRAJECTORY OPTIMIZATION METHODS AND TOOLS

The basic optimal control problem involves determining the control vectors and parameters to minimize some performance index[9]. As mentioned earlier, the performance index to be minimized in this case is the propellant consumption. The control vector, specified in Eq. (1) as $\bar{u}_c$, consists of the thrust-direction unit vector, the thrust magnitude (T) and the power (P), which is required for variable specific impulse engines. The general trajectory equations of motion are also given by Eq. (1).

$$\dot{X} = \begin{bmatrix} \dot{\bar{r}} \\ \dot{\bar{v}} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \dot{\bar{v}} \\ -\left(\mu / r^3 + (T/m)\bar{u}\right) \\ -(T/c) \end{bmatrix} \qquad \bar{u}_c = \begin{bmatrix} \bar{u} \\ T \\ P \end{bmatrix} \tag{1}$$

Additionally, for each leg of the trajectory, the spacecraft's initial conditions must match the position and velocity of the point of origin. For a flyby trajectory, the spacecraft's final conditions must match the target's position, and for a rendezvous trajectory, the spacecraft must match the target's position and velocity. For a rendezvous mission leg, these physical state constraints are given by Eq. (2).

$$\psi = \begin{bmatrix} \bar{r}_{s/c}(t_0) - \bar{r}_t(t_0) \\ \bar{v}_{s/c}(t_0) - \bar{v}_t(t_0) \\ \bar{r}_{s/c}(t_f) - \bar{r}_t(t_f) \\ \bar{v}_{s/c}(t_f) - \bar{v}_t(t_f) \end{bmatrix} \tag{2}$$

Additionally, there are constraints on maximum thrust and power, dependent on the engine parameters chosen for the spacecraft.

In general, there are two types of methods for solving the local trajectory optimization problem – direct and indirect[10,11,12,13]. Indirect methods, based on calculus of variations, formulate the optimal control problem as a two-point boundary value problem, which is solved by satisfying terminal conditions and targeting constraints. If a solution to the TPBVP is obtained, the resulting trajectory is the optimal solution for those particular initial conditions and targeting constraints. Finding a solution, however, is often difficult because the convergence domain for such problems tends to be small, and is very sensitive to the initial guesses of the costate variables, which are generally not physically intuitive. Adding intermediate gravity assists further increases the sensitivity to the initial guesses and further decreases the convergence domain. In order to solve these problems, a homotopy chain is often used, where the solution to a similar problem is known, and that problem is changed slightly and solved with the initial guesses of the known problem in order to step closer to the problem of interest[14]. Therefore, typical indirect methods are difficult to implement within an automated, global optimization program due to the long execution times, small region of convergence, and required user oversight. Additionally, the level of accuracy achieved by indirect methods is not required during the conceptual mission design phase.

Direct methods, on the other hand, parameterize the optimal control problem and use nonlinear programming (NLP) techniques to directly optimize the performance index. A variety of different direct method exists, including collocation, direct transcription, and differential inclusion[9,15,16,17,18]. The number of design variables for direct methods can become very large, and therefore these problems are limited by current NLP techniques. Additionally, because direct methods require discretization of a continuous problem, the solution is considered sub-optimal, although the accuracy is generally sufficient for conceptual design. The main advantages of direct method techniques are their increased computational efficiency and their more robust convergence. The solution is less sensitive to the initial guesses and those initial guesses are more physically intuitive, which make direct methods preferable for implementing within an automated global optimization scheme.

Finally, there also exist hybrid methods[12,19], which numerically integrate the Euler-Lagrange equations and control the spacecraft based on the primer vector. As in the direct method, the hybrid methods solve a nonlinear programming problem, but with the Lagrange multipliers making up part of the parameter vector while maximizing or minimizing some cost function. Hybrid methods search numerically for the set of parameters that extremize the cost function, while explicitly satisfying only the kinematic boundary constraints.

Low-Thrust Trajectory Optimization Tools

There are a wide variety of available tools for low-thrust trajectory optimization, many based on the methods described above. In 2002, NASA established the Low-Thrust Trajectory Tools Team (LTTT) to improve the agency’s low-thrust trajectory analysis capability and to create a common set of low-thrust trajectory tools[8,20]. Under the effort, five new tools were developed, and 32 reference missions were identified that would be relevant to future NASA missions and would test the capabilities of these new tools. The reference missions include numerous missions with multiple gravity assists as well as flybys of and rendezvous with comets and asteroids. In general, the new tools are of higher fidelity, easier to learn and use, and can analyze a broader range of missions than the previously existing set of tools.

Prior to the LTTT effort, the primary low-thrust trajectory analysis tools for most of NASA’s preliminary design studies were CHEBYTOP, VARITOP, SEPTOP, and SAIL[8]. CHEBYTOP uses Chebychev polynomials to represent state variables, which are then differentiated and integrated in closed form to solve a variable-thrust trajectory. This solution can then be used to approximate a constant thrust trajectory. While it is considered a low-fidelity program, it is highly valued for its ability to rapidly assess large trade spaces. It cannot, however, analyze multi-leg missions and is limited to the heliocentric sphere of influence. VARITOP, SEPTOP, and SAIL all use calculus of variations in the formulation of the state and co-state equations, which are integrated numerically to solve the two-point boundary value problem. The programs differ in their solar electric propulsion, nuclear electric propulsion, and solar sail models. In general, these tools can also only handle heliocentric trajectories, and are considered to be medium-fidelity.

The tools developed under the LTTT effort are all considered to be medium- to high-fidelity trajectory tools[8]. MALTO was developed at JPL based on a method by Sims and Flanagan, which will be explained later in the paper. Basically, this tool implements a direct method, simulating a continuous burn trajectory with a number of impulsive burns, around a single gravitational body. It is considered to be medium fidelity. This tool has been used for numerous trajectory design studies, including the design of the Jupiter Icy Moons Orbiter. The remaining LTTT tools are all considered to be high-fidelity. Copernicus, developed at the University of Texas at Austin, is an n-body tool with a high degree of flexibility. The user can model a number of different missions, with varying gravitational bodies, objective functions, optimization variables, constraint options, and levels of fidelity. Additionally, it can model multiple spacecraft, as well as optimize for both constant and variable specific impulse trajectories. COPERNICUS employs multiple shooting and direct integration for targeting and state propagation[21]. Mystic was developed by Greg Whiffen at JPL, and implements Static/Dynamic Optimal Control (SDC), which was developed by the author. SDC is a nonlinear optimal control method designed to optimize both static variables and dynamic variables (functions of time) simultaneously[22]. One of the main strengths of this tool is its ability to automatically find and use gravity assists if beneficial to the trajectory. Mystic is being used to design the Dawn trajectory, and after being flight qualified, is expected to be used to validate the other tools. OTIS 4.0 is an upgraded version of the program originally developed by NASA Glenn Research Center and Boeing for launch vehicle trajectory analysis[23,24]. This tool employs a direct method for low-thrust trajectory optimization, using nonlinear programming techniques to solve the implicit integration problem. SNAP, developed at NASA Glenn Research Center, is the final tool developed under the LTTT effort. SNAP's distinguishing feature is its ability to propagate planet-centered trajectories, including aspects such as atmospheric drag, shadowing, and higher-order gravity models. It does not, however, contain an optimizer.

With the exception of SNAP, the various tools described above were compared for a number of different low-thrust mission scenarios. Ref. 8 provides an overview of five of the 32 reference missions examined, and compares in detail the results of the various tools. In general, however, it was found that the low, medium, and high fidelity tools arrived at very similar answers when their input assumptions were consistent. The high fidelity tools do not necessarily provide significant improvements in accuracy, but are able to model more complex missions. Low fidelity tools, on the other hand, have the advantage of faster execution times, rapid trade study analysis, and are often much easier to learn and implement. Therefore, for application to the global optimization of a multiple asteroid tour mission, a low fidelity tool may be the most appropriate, if it can be extended to model such a complicated mission.

In addition to the LTTT tools, several recent university-developed tools have been created for low-thrust trajectory optimization. Petropoulos at Purdue University incorporated a low-thrust gravity assist capability to STOUR (Satellite Tour Design Program) to create STOUR-LTGA, which automatically searches for gravity-assist trajectories[25]. In this program, the user specifies a sequence of gravity-assist bodies, a range of launch dates, a range of V_∞s, and constraints on various parameters, such as time of flight and propellant consumption. STOUR-LTGA employs a shape-based method to approximate the shape of the trajectory and analytically solve the equations of motion. This method will be described in more detail later in the paper. Also developed at Purdue University, GALLOP implements the direct method formulated by Sims and Flanagan, which is also found in MALTO[26,27]. More detail on both methods as well as results generating using these tools will be presented in subsequent sections.

Recent Improvements to Indirect Methods

In general, tools using indirect methods for trajectory optimization are considered high-fidelity tools because they numerically integrate the equations of motion and determine the continuous control history of the spacecraft. As a result, however, they generally require long run times are not suited for conceptual design, when a broad exploration of the trade space is required. Additionally, as aforementioned, another obstacle to their use during conceptual design is the need for user oversight, because of the small convergence domain and the need for non-intuitive initial guesses. The problem of non-intuitive initial guesses has been recently addressed by Ranieri at the University of Texas at Austin, who employs an adjoint control transformation to give physical meaning to the initial estimates of the costate vector[12,28]. Ranieri applies this technique to solving a roundtrip, time-constrained trajectory with I_{sp} constraints and mass discontinuities, which has many similarities to an asteroid rendezvous sample return problem. In fact, this method can be directly applied to a roundtrip trajectory to a single asteroid, but would need to be modified to include multiple asteroid rendezvous. Figure 4 presents a schematic of the mission of interest.

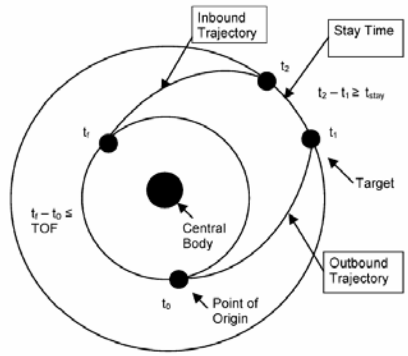

Figure 4 Schematic of a roundtrip, time-constrained trajectory.

In solving the optimal control problem as an indirect optimization problem, the Euler-Lagrange equations are integrated numerically, with the thrust direction and constrained engine parameters comprising the controls. The optimization problem is to minimize propellant consumption and is formulated as either a maximization of the final mass for a given initial mass or a minimization of the initial mass for a given final mass. The start and end times of the trajectory are free search variables, although the solution will be a local minimum based on the initial guesses. The resulting multipoint boundary value problem is then solved using the controls based on the Pontryagin maximum principle. Instead of requiring initial guesses for the costates, however, the adjoint control transformation allows the velocity costates to be replaced with angles that describe the direction of the thrust. These new unknowns have actual physical significance; therefore intelligent estimates of their initial guesses can be made.

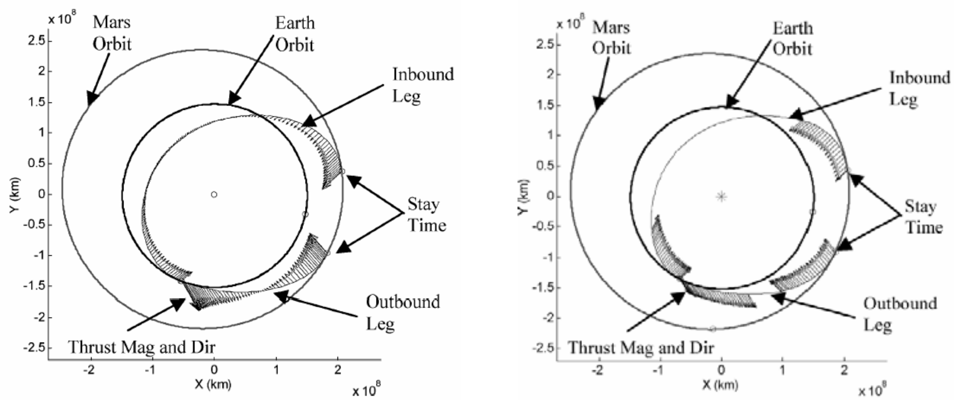

Figure 5 VSI roundtrip to Mars departing 20 May 2018 (left); CSI roundtrip to Mars departing 24 May 2018 (right).

Ranieri applies this methodology to roundtrip trajectories to both Mars and Jupiter, for variable specific impulse (VSI) and constant specific impulse (CSI) engines. Two cases for roundtrip trajectories to Mars are presented in Figure 5. For the CSI case, a coast-thrust-coast sequence is assumed for each leg of the trajectory. As can be seen, the CSI trajectory closely approximates the VSI solution.

Recent Improvements to Direct Methods

MALTO, the tool mentioned above that was created under the recent LTTT effort, is based on a direct method by Sims and Flanagan[10,11]. This tool is intended to be used primarily for preliminary design of low-thrust interplanetary trajectories including those with multiple gravity assists. Figure 6 illustrates the structure of the trajectory used in this method. As shown, the trajectory is divided into legs that begin and end at control nodes. Typically, these control nodes represent planets or other bodies, but could also represent free points in space. On each leg is a match point, and the trajectory is propagated forwards from the previous control node and backwards from the subsequent control node to the match point. The purpose of employing this multiple shooting technique is to reduce the sensitivity of the propagation to intermediate flybys. Each leg is also subdivided into numerous segments containing an impulsive ΔV at the middle of each segment, in order to approximate a continuous thrust problem. The magnitude of this ΔV is limited by the total amount of ΔV that could be accumulated over the entire segment for the continuous thrusting case. The propagation of the trajectory assumes the two-body problem, and gravity assists are assumed to cause an instantaneous change in the direction of the V_∞ vector.

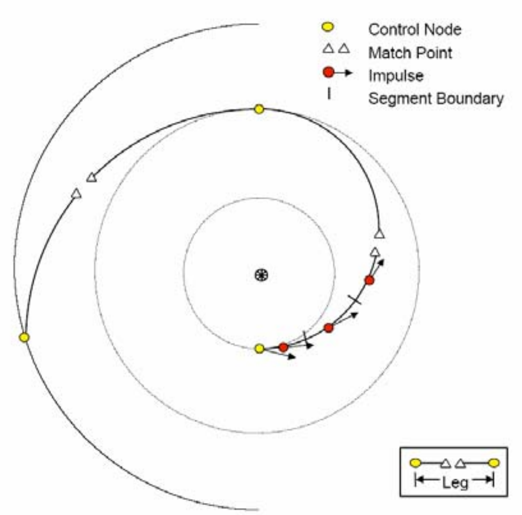

Figure 6 Trajectory structure of the Sims and Flanagan direct method.

This trajectory structure leads to a constrained, nonlinear optimization problem, which is solved using the program SNOPT developed at the University of California San Diego. MALTO uses analytic derivatives (instead of finite differencing), which contribute largely to the fast execution and robust convergence of the tool. The problem formulation, however, leads to a large number of independent variables. At the beginning and ending control nodes, the independent variables include the velocity of the spacecraft relative to the body, the mass of the spacecraft, and the corresponding epoch. At an intermediate body, there are two sets of variables – one at arrival and one at departure – to account, for example, for changes in velocity for a flyby, changes in mass, or changes in time for a rendezvous. The majority of the independent variables are comprised of the components of the thrust vector on each segment. Additional independent variables can include the reference power of the spacecraft and the specific impulse for a variable specific impulse trajectory. All of these independent variables have associated upper and lower bounds. The primary optimization constraints are that the position, velocity, and mass of the spacecraft must be continuous at the match points. Additionally, the magnitude of the thrust on each segment is constrained by the power available for thrusting. Other constraints can include the mass at the initial control node, the V_∞ vector at departure, at an intermediate body, or at arrival, the time of flight and propellant mass between any two control nodes, and the minimum allowable distance from the Sun. In implementing this method in MALTO, the following optimizations were enabled: maximize final spacecraft mass, minimize initial spacecraft mass, minimize total trip time, optimize a weighted combination of final mass and trip time, and maximize the Earth miss distance of a small body after impact.

In the original paper by Sims and Flanagan, the authors applied their direct method to several different trajectories, verifying their results by comparison to SEPTOP: a flyby of the asteroid Vesta with a Mars gravity assist, a rendezvous with the comet Tempel 1, and a flyby of Pluto with two Venus gravity assists and one Jupiter gravity assist. With their direct method, even simple initial guesses for thrust direction and magnitude worked well in arriving at the solution. For the initial guess, they assume that the thrust varies linearly between nodes, with the direction at the nodes being perpendicular to the radius vector at that point. The solutions for the three reference missions compared very well to those obtained using SEPTOP. For the Vesta and Tempel 1 trajectories, SEPTOP actually had difficulty converging for some of the cases, while the Sims and Flanagan method converged readily. Furthermore, SEPTOP could only handle at most two intermediate flybys, so the Earth-Venus-Venus-Jupiter-Pluto trajectory had to be broken into two trajectories in SEPTOP. Using the Sims and Flanagan method, any number of intermediate bodies can be analyzed. They do note that for more complicated trajectories, however, the optimization does not always converge with the initial starting conditions, so a fair amount of user manipulation is still required to arrive at a converged solution.

In addition to the reference missions analyzed by Sims and Flanagan, several papers out of Purdue University include additional results using GALLOP, an in-house trajectory optimization tool based on the Sims and Flanagan direct method[29,30]. These additional trajectories include a rendezvous with Ceres via Mars, an Earth-Venus-Earth-Mars-Jupiter trajectory, an Earth-Venus-Jupiter trajectory, an Earth-Mars-Jupiter trajectory, and an Earth-Earth-Mars-Jupiter trajectory. This helped to further validate the method as well as demonstrate its ability to handle a number of different flyby problems with numerous intermediate bodies. For the Purdue studies, however, the initial guesses were generated by a shape-based analytic method (described later in the paper).

Building on the Sims and Flanagan method, Yam at Purdue University explored different formulations for parameterizing the $\Delta\mathbf{V}$ in an effort to decrease run time[26,27]. The formulations examined are as follows:

1. N-Vector Formulation (original method used in Sims and Flanagan model) – The optimization variables consist of the $\Delta\mathbf{V}$ coordinates on each segment (ΔV, θ, ψ) or (ΔV_x, ΔV_y, ΔV_z), with the maximum allowable ΔV constraint explained above. For n segments, this results in $3n$ variables and n nonlinear constraints.

2. Node (On/Off-Node) Formulation – This formulation replaces the n $\Delta\mathbf{V}$ magnitudes with a set of On/Off nodes. Here, an off-node defines the switching point from maximum-thrust (MT) to null-thrust (NT) and an on-node defines the switching point from null-thrust to maximum-thrust. Constraints can be placed so that the order of the nodes can be pre-specified (e.g., thrust-coast-thrust). There are eight optimization variables for each on/off-node: position, velocity, mass, and time, resulting in a total of $8k_N + 2n$ optimization variables for k_N nodes.

3. Chebyshev Formulation – For each leg, the $\Delta\mathbf{V}$ angles (θ and ψ) are modeled as a Chebyshev series:

$$c_0 T_0(u) + c_1 T_1(u) + \cdots + c_k T_k(u) \tag{3}$$

   The parameters associated with the $\Delta\mathbf{V}$ angles are the coefficients on the Chebyshev series, $\{c_0, c_1, \ldots, c_k\}$. The number of variables for this formulation is $n + k_\theta + k_\psi + 2$, where k_θ and k_ψ are the degree of the Chebyshev series on θ and ψ.

4. Node + Chebyshev Formulation – This combines the Node formulation and the Chebyshev formulation. The on/off-nodes are used to parameterize the $\Delta\mathbf{V}$ magnitude, while the $\Delta\mathbf{V}$ angles are modeled by a Chebyshev series. Therefore, for this formulation, the number of variables required to parameterize the $\Delta\mathbf{V}$ is $8k_N + k_\theta + k_\psi + 2$.

In Ref. 27, four different case studies are examined to determine the performance of each of the four $\Delta\mathbf{V}$ parameterizations. For a simple Earth-Jupiter rendezvous mission, the Node + Chebyshev formulation had a significantly faster run time than the other formulations. The Chebyshev formulation had convergence difficulties for certain starting times of flight, in that the solution would converge to a suboptimal solution (local minimum). The second mission examined is a flyby of the asteroid Vesta, with a Mars gravity assist. Several different cases were run, with the following parameters beginning as fixed and then all set as free variables: V_∞, Earth launch date, Mars flyby date, and Vesta arrival date. When the four parameters were free search variables, the Chebyshev formulation had the fastest run times. The worst performing formulation in this case was the Node + Chebyshev formulation, which is almost ten times slower. For the other three variations on fixed and free parameters, however, while the Chebyshev formulation performed well, the Node + Chebyshev actually had the fastest run times. The third mission considered was an Earth-Mars cycler, which transfers back and forth between Earth and Mars once every synodic period for 15 years. This is a very large optimization problem that includes 15 planetary encounters and 371 segments in total. For the original N-Vector formulation, this resulted in over 1000 variables and almost 500 nonlinear constraints. The Chebyshev and Node + Chebyshev had problems with convergence using the default tolerances of 10^{-6}. If this tolerance were relaxed to 10^{-4}, the Node + Chebyshev formulation had significantly faster run times than the other formulations, and the final feasibility is actually on the order of 10^{-10}. The N-Vector and Node formulations took 15 and 20 hours, respectively, to arrive at the solution. The final mission examined was an Earth-Mercury rendezvous mission, which requires many revolutions around the sun. As a result, this problem also involved a large number of design variables and constraints. In this case, the N-Vector had the fastest run times, although the Chebyshev formulation also performed well. The Node + Chebyshev also had good performance but again only if the tolerances were relaxed slightly.

From these results, it is clear that the best formulation to use is very problem dependent, although time savings can be realized over the original N-Vector formulation developed by Sims and Flanagan. The Node + Chebyshev formulation tended to have the fastest run times for the largest range of problems, however, it did have problems with convergence. For large problems, the tolerances had to be relaxed in order for the Node + Chebyshev formulation to arrive at a solution. This method is the most beneficial for searching broad areas of the design space. On the other hand, the N-Vector formulation is the most stable, although it was not always the fastest approach, and in some cases, it was significantly slower. It is a good standard method when only a small number of cases need to be performed.

Shape-Based Analytic Methods

Indirect and direct methods tend to be computationally intensive because the trajectory must be propagated or numerically integrated. An analytic method, on the other hand, has the potential to significantly reduce run times by eliminating the need for numerical integration and instead solving for an analytic solution to the equations of motion. Petropoulos, at Purdue University, developed a shape-based method intended for quickly searching a broad design space and generating initial guesses to then be used in a more accurate trajectory optimization program[25,31,32]. This method is implemented within STOUR-LTGA, which is meant for the design of low-thrust gravity assist trajectories.

This method assumes that the spacecraft trajectory follows a predetermined shape, from which the thrust profile can be determined. With the correct choice of shape, there exists an analytic solution to the equations of motion. The motion of the spacecraft between planets can either be purely conic (coasting) or involve thrusting. Each leg can be characterized as thrust, thrust-coast, or coast-thrust. For the thrusting segments, the in-plane motion of the spacecraft is assumed to follow an exponential sinusoid shape, given by Eq. 4, where k_0, k_1, k_2, and ϕ are all constants.

$$r = k_0 e^{k_1 \sin(k_2\theta+\phi)} \tag{4}$$

Gravity assists are modeled as instantaneous changes in the heliocentric spacecraft velocity with no change in position. The out-of-plane motion is based on an analysis of the orbital angular momentum vector, where the out-of-plane angle and speed are approximated by the in-plane angular momentum and velocity components.

This method has been applied to a number of different trajectories[25,29,30,32]. One is a rendezvous with the asteroid Ceres with an intermediate flyby of Mars, assuming thrust-only legs. A search was done for departure dates ranging from 1990 to 2049 with launch V_∞s between 0.75 km/s and 2 km/s. Figure 7 (left) plots the resulting arrival V_∞ for each of the cases analyzed. This broad search allows mission designers to choose the best points to examine further with higher-fidelity trajectory optimization methods. In this study, the best point from the shape-based analysis was then used as an initial guess for GALLOP, a trajectory optimization code based on the direct method by Sims and Flanagan. The result has good agreement with an optimal solution presented by Sauer in an earlier study. Another trajectory analyzed by Petropoulos is Earth-Venus-Earth-Mars-Jupiter (EVEMJ). One of the strengths of STOUR-LTGA is its ability to model trajectories with many legs. A sweep of departure dates from 1975 to 2049 was analyzed, with an increment of 10 days (over 2700 different launch dates!). Additionally, launch V_∞s between 0.5 km/s and 2 km/s were considered, with a maximum time of flight of 2500 days and maximum times of flight set for each leg. Mars-Jupiter is assumed to be a thrust-coast leg, while the other three legs are thrust-only. For this case, the in-plane propellant mass fraction is the parameter of interest, which is plotted in Figure 7 (right). As before, the best trajectory from this broad design space exploration was used as an initial guess in GALLOP in order to optimize the solution.

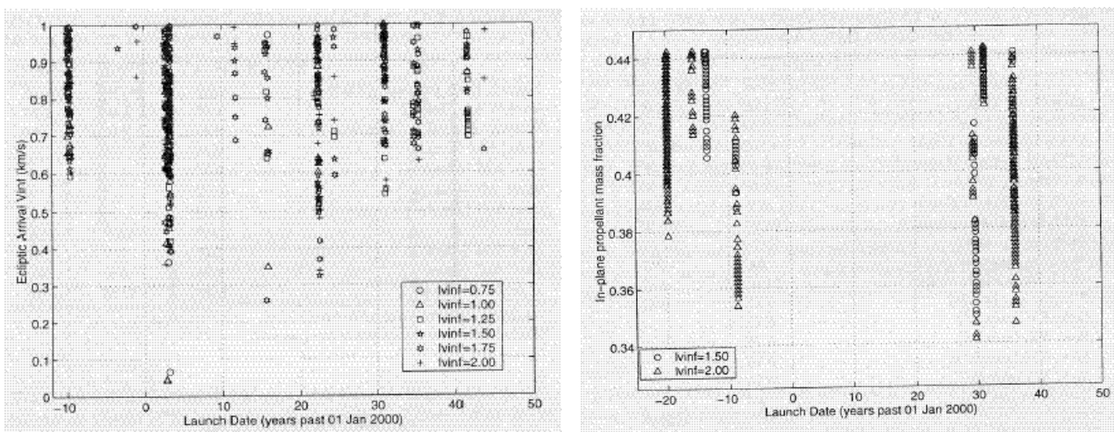

Figure 7 Results of STOUR-LTGA design space exploration; arrival V∞ for a range of launch dates for Earth-Mars-Ceres (left); in-plane propellant mass fraction for a range of launch dates for EVEMJ (right).

This method was also applied by the winning team, a team from NASA's Jet Propulsion Laboratory (JPL), at the 2006 1[ST] Advanced Concepts Team Global Trajectory Optimisation Competition, sponsored by the European Space Agency[33]. The objective of the optimization problem was to maximize the change in the semi-major axis of asteroid 2001 TW229 after impacting it with an electric-propelled spacecraft. The initial mass of the spacecraft was given, along with the thruster's I_{sp} and maximum thrust level. Additionally, a span in launch date of 20 years was given, with a maximum time of flight of 30 years[34]. In approaching this problem, the team from JPL took a two-step approach[35]. First, they searched over a large range of the solution space, then honed in on the most promising portion with a local optimization method. The JPL team considered 15 different gravity assist combinations, and then conducted a grid search for each combination over launch date and launch V_∞ values using STOUR-LTGA. The best solutions from the grid search (high values of arrival V_∞) were then passed on to MALTO to examine in more detail. The optimal trajectory departed Earth on 8/20/2024, following a VEEEJSJ sequence, before impacting the asteroid with an arrival V_∞ of 52.662 km/s and an impact mass of 1442.91 kg (the only thrust leg was the first Earth-Venus leg).

GLOBAL OPTIMIZATION METHODS AND APPLICATIONS

Many of the tools described above not only implement a trajectory optimization method for finding the optimal control history of the spacecraft (thrust magnitude and direction), but also include some ability to optimize for other parameters such as launch date or arrival date. As explained earlier, however, if a gradient-based optimizer is used, the resulting solution will likely be a local optimum and not the global optimum. Because the design space is multi-modal, the solution will be highly dependent on the initial guess of your parameters. If a broad search space is desired, such as in the case of the STOUR-LTGA examples, a domain-spanning, global optimization method is required. This section of the paper will survey the most common global optimization methods and their application to low-thrust trajectory optimization problems.

Common Global Optimization Methods

One of the most well known types of global optimization methods are called evolutionary algorithms, which are domain spanning, probabilistic optimization algorithms based on the Darwinian theory of evolution[36]. One of the more well known of these evolutionary algorithms is the genetic algorithm (GA)[37,38]. Although there are numerous variations, the general genetic algorithm begins with a random initial population, which is made up of multiple sets of values for each of the design variables. Each member of the population represents a single value for each of the design variables. This generally results in a random scatter of points over the entire design space. Each set of design variables is referred to as a chromosome and is typically encoded as a binary string, which must be mapped to the real values of the variables, although there are a number of different approaches to the encoding process. The design variables are discretized between their lower and upper bounds. In each generation, the population undergoes certain genetic operators such that the population will "evolve" and improve its fitness (objective function). The typical genetic operators are reproduction, crossover, and mutation. The purpose of reproduction is to weed out the members of the population with low fitness, and to keep those with high fitness. Crossover combines two "parents" by switching parts of their chromosome strings with each other, while mutation is responsible for switching individual bits in a chromosome string. Figure 8 illustrates a schematic of the GA process. Because there is no necessary condition for optimality, the convergence criteria is usually chosen either as a maximum number of generations (iterations) or a certain number of generations with no change in the objective function. As the generations progress, there should be a steady improvement in the both the average fitness of the population as well as the fitness of the best member. In general, at the termination of the GA, all of the members of the population will be clustered around the global optimum.

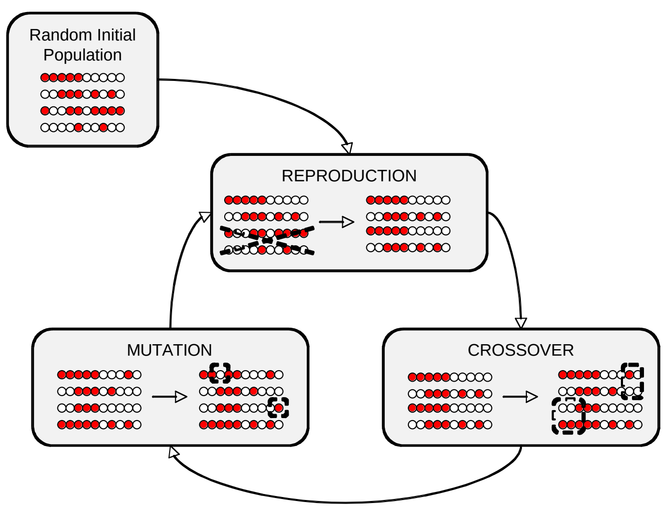

Figure 8 Schematic of a genetic algorithm.

One of the main advantages of genetic algorithms is their ability to find a global optimum in a multi-modal design space. They can also handle a large number of variables, and require no initial guesses for the design variables. They do, however, have some downfalls. Because of the probabilistic nature of the algorithm, there is no guarantee that the optimal solution will be arrived at. Therefore, the GA must generally be run more than once to ensure optimality. Genetic algorithms also require a large number of iterations, and therefore function calls, in comparison to a gradient-based method. Finally, because the design space is being discretized, the solution will generally not correspond to the global optimum of a continuous problem. A common practice is to then run a gradient-based optimizer using the solution found by the GA.

Another global optimization technique, which has just recently been applied to trajectory optimization problems, is the Evolutionary Neurocontroller (ENC)[39,40,41], which combines artificial neural networks (ANNs) with evolutionary algorithms. Artificial neural networks are inspired by information processing in animal nervous systems, in that they will learn from experience, generalize previous examples to new ones, extract essential information from noisy input data, etc. ANNs are composed of processing elements called neurons that are organized into neuron layers. Figure 9 illustrates an example of a feedforward ANN, with a layered topology and three layers.

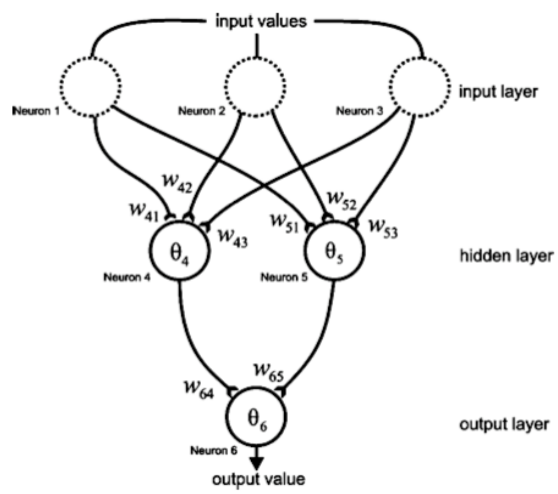

Figure 9 Example of a layered, feedforward neural network.

Depending on the function used for the neurons, a neural network can be regarded as a continuous parameterized function, called a network function, which simply maps a set of inputs to a set of outputs. If a training set exists – the correct output for a set of given inputs – then the network error can be measured and used to learn the optimal network function. If a training set does not exist, then it is a reinforcement learning problem, where the optimal behavior must be learned through interaction with the environment. For an evolutionary neurocontroller, the evolutionary algorithm is instead used to find the neurocontroller's optimal network function.

Application of Global Optimization Methods to Low-Thrust Trajectory Optimization Problem

Genetic algorithms have long been applied to a number of different problems, including trajectory optimization problems, beginning with their application to ballistic (high thrust) gravity assist problems[42,43,44,45]. For the high-thrust case, solving for a single trajectory is much less time-consuming and is generally done using a Lambert Solver. Therefore, a genetic algorithm, even with a large number of required function calls, is appropriate for global optimization. Several studies have also attempted to apply genetic algorithms to solve for the optimal control parameters in the low-thrust problem[46,47]. This approach, however, has not shown any benefits over direct or indirect methods for trajectory optimization, again because of the large number of function calls required by the GA. More recently, several authors have attempted to apply the genetic algorithm to selecting the global parameters of the optimization problem, combined with a direct or indirect method for solving for the control history of the spacecraft[48,49,50].

De Pascale proposes a method for combining a genetic algorithm with an analytic shape-based method to optimize low-thrust gravity assist trajectories[48]. The trajectory is divided into sub-arcs, which are chosen to be either coast arcs or low-thrust arcs. The two-point boundary value problem for the coast arcs is solved simply with a Lambert solver, while the low-thrust arcs are solved using a shape-based method based on the work by Petropoulos. For this work, an exponential trigonometric shape is used to analytically solve the equations of motion. Gravity assists are modeled as instantaneous changes in the heliocentric velocity. The genetic algorithm is used in conjunction with a static penalty function, in order to handle constraints. The full set of design variables is given by Eq. 5, and includes the departure V_∞, the right ascension and declination at launch, the velocities at each of the encounter bodies, the sequence of encounter bodies, the pericenter-radius for each flyby, and the number of revolutions around the Sun for each phase.

$$y = \left[v_\infty^E, \psi, \vartheta, v_{in}^1, \ldots, v_{in}^{N_p-1}, t_1, \ldots, t_{N_p}, p^{(1)}, \ldots, p^{(N_p-1)}, \tilde{r}_p^{(1)}, \ldots, \tilde{r}_p^{(N_p-1)}, n^{(1)}, \ldots, n^{(N_p-1)}\right]^T \tag{5}$$

The method proposed by De Pascale was applied to several different trajectories. First, a simple low-thrust transfer to Mars was examined. The solutions obtained matched very closely to existing optimal solutions for this problem. Ballistic (high-thrust) missions to Jupiter were then examined, using the full set of design variables, so that the gravity assist sequence was not predetermined. Several promising trajectory paths resulted: EVEEJ, EMMJ, and EVVEJ. When low-thrust trajectories to Jupiter were considered, however, the author did not use the full set of design variables, but instead optimized the trajectory for predetermined sequences of gravity assists (EVJ, EVVJ, and EMMJ). It was not clear if the method had failed for the full set of design variables in the low-thrust case or if it simply had not been attempted.

Woo, Coverstone, and Cupples proposed a method combining a genetic algorithm with SEPTOP, which uses an indirect method for solving the optimal control problem[49]. One of the key features of this work is the procedure for reducing the size of the parameter space before applying the GA/SEPTOP hybrid method. Trajectories previously generated by SEPTOP are used to limit the size of the design space through a number of different methods: R-ratio analysis, delivered mass estimation, thruster modeling, ballistic approximation, and phase calculation. More detail on each of these methods can be found in Ref. 49. The genetic algorithm is then used to search the reduced parameter space, which generates inputs to run SEPTOP. SEPTOP returns the convergence error to the genetic algorithm as a measure of the fitness of the initial input. Results are generated for a series of outer-planet missions with a single Venus gravity assist. In previous work, this hybrid procedure was also successfully applied to the design of a trajectory for a sample return to the comet Tempel 1[51]. The reduction of the parameter space, however, could not be applied because there were no previously generated trajectories.

Finally, the Japan Aerospace Exploration Agency proposed a multiple asteroid sample return mission as a possible follow-on mission to MUSES-C[50]. For this trajectory design, twenty asteroids were selected as potential targets, from a deliverable mass point of view. Three possible trajectory sequences were analyzed: Earth->Asteroid 1->Asteroid 2-> Earth, Earth->Asteroid 1->Earth swing-by (n-times)->Asteroid 2->Earth, and Earth->Earth swing-by->Asteroid 1/Asteroid 2->Earth (multiple spacecraft). In Ref. 50, the authors claim that a genetic algorithm was used to select the target asteroids, and dates of departure and arrival, as shown in Figure 10, but no details of the method are given.

Figure 10 Parameters for genetic algorithm optimization in JAXA study.

Evolutionary neurocontrol has also been recently applied to the low-thrust trajectory optimization problem [39,40]. Bernd Dachwald, with the Institute of Space Simulation in Germany, originally applied the evolutionary neurocontrol method to solar sail trajectories, which have thrust magnitudes much smaller than a typical electric propulsion spacecraft, thereby exhibiting very different solutions with many revolutions around the Sun. Furthermore, the objective function is generally minimizing time of flight since there is no propulsion required for a solar sail. More recently, however, Dachwald applied his method to solar electric propulsion (SEP) spacecraft. In his formulation, a trajectory is the result of a spacecraft steering strategy that controls the spacecraft's thrust vector according to the current state of the spacecraft relative to the target. An artificial neural network is then used to implement the spacecraft steering strategy, with the evolutionary algorithm used to optimize the neurocontroller parameters. Figure 11 illustrates how such a formulation works for the SEP trajectory. The neural network pictured below illustrates how the inputs for a SEP trajectory are mapped to outputs, as per Dachwald's formulation. Here, the inputs represent the difference in the spacecraft's state and its target at any point along the trajectory. The output then corresponds to the control parameters that will result in the spacecraft meeting its target constraints at the specified final time.

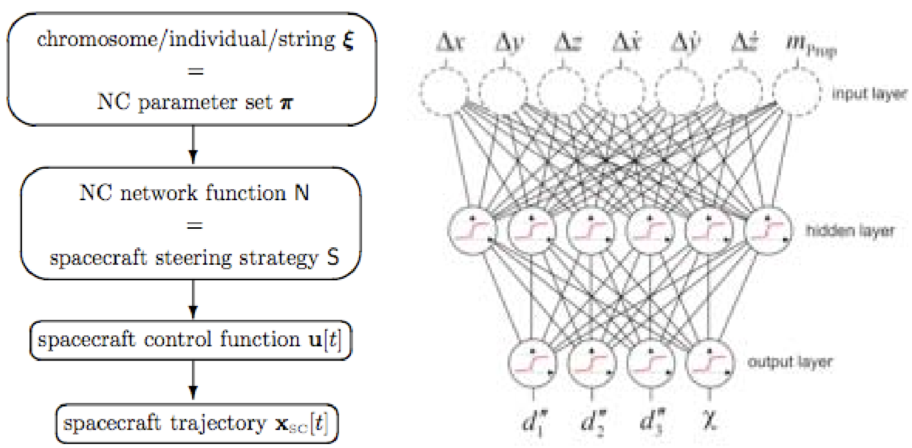

Figure 11 Converting an evolutionary algorithm chromosome into a spacecraft trajectory (left); example neurcontroller that implements a spacecraft trajectory (right).

Dachwald utilizes the evolutionary neurocontroller to optimize the launch date for his example cases, in addition to the spacecraft steering strategy. He does not, however, consider problems with multiple legs or encounter bodies. The evolutionary neurocontroller was applied to a Mercury rendezvous and a near-Earth asteroid rendezvous, and compared to similar problems in the literature. Dachwald's method was able to locate solutions better than those presented in the literature, due to its ability to exhaustively search the design space.

Carnelli later extended Dachwald's method to include low-thrust trajectories with gravity assists[41]. An evolutionary neurocontroller is combined with a steepest descent method used to optimize the gravity assist maneuvers. As before, the ENC searches for the optimal parameter set (steering strategy) that forces the spacecraft's state from its initial state to the target body's final state, along a trajectory that obeys the dynamic constraints and terminal constraints, while maximizing some cost function and potentially crossing the sphere of influence (SOI) of an assisting body. Instead of choosing some sequence of gravitational assists *a priori*, the ENC is freely allowed to choose the spacecraft controls, and a gravity assist is performed only if that steering strategy takes the trajectory through the SOI of some intermediate planet. Because the relative size of the SOIs is very small in comparison to the scale of the overall trajectory, their size had to be inflated. Otherwise, the ENC would be very unlikely to ever find a gravity assist trajectory. When the chosen steering strategy does take the spacecraft within a planet's SOI, a steepest-descent algorithm is used to determine the optimal pointing distance for the gravity assist maneuver. Making these modifications allowed Carnelli to successfully apply this method to a Pluto flyby trajectory via Jupiter and a Mercury rendezvous via Venus.

CONCLUSIONS

This paper posed a challenging trajectory optimization problem: the global optimization of a low-thrust, multiple asteroid tour mission, which requires both global, domain-spanning optimization, and local trajectory optimization of the spacecraft control history. Full design space exploration is desired during the conceptual design phase, when some degree of accuracy can be sacrificed to achieve faster execution times. For each function call of the global optimizer – for a set of values for launch date, times of flight, sequence and timing of intermediate body encounters, Earth return date, etc. – the local trajectory optimization must also be run. A number of different trajectory optimization methods are available, many of which were addressed in this paper, including direct, indirect, hybrid, and analytic methods. Although advances have been made to indirect methods to reduce the degree of required user manipulation, these techniques are generally still too computationally intensive to be implemented within an automated global optimization context. Additionally, the resulting accuracy of the results is not required, particularly when lower fidelity approaches yield similar results. Direct methods are more promising because of their more robust convergence and faster run times, particularly with the findings of Yam and Longuski on methods for further decreasing run times through different parameterizations of the ΔV. Shape-based methods, however, appear to be the most promising for application to such a large global optimization problem. Because they result in an analytic solution to the equations of motion, a local optimization does not need to be conducted for each step of the global optimizer. The method developed by Petropoulos has been successfully applied to numerous different case studies, to provide a handful of initial guesses for a higher fidelity trajectory optimization code.

Many problems similar to the multiple asteroid tour, generally involving trajectories with multiple gravity assists, have been tackled using a variety of the above methods. In most cases, however, the choice of gravity assist planets or sequence of asteroid encounters is predetermined and then the trajectory is optimized for that particular sequence over a large range of launch opportunities, values of V_∞, times of flight, and so forth. The 1st ACT Global Trajectory Optimisation Competition challenged teams to find the global optimum to intercept an asteroid, but without a predetermined sequence of gravitational assists for a large range of launch dates. Even the winning JPL team did not fully automate this process, but instead determined fifteen different sequences to further examine using STOUR-LTGA. An exhaustive search was then done individually for each of the fifteen cases. Therefore, it is still possible that a better solution was missed because the corresponding gravity assist sequence was not one of those considered. Future work could integrate STOUR-LTGA, or a similar method, with an evolutionary algorithm, where the sequence of intermediate bodies is included as a free search variable in order to more fully explore the design space. Carnelli's approach to using evolutionary neurocontrollers to tackle a low-thrust, gravity assist problem is also promising. The results presented, however, only included a single gravity assist, and more work would be required to determine how the method would extend to finding multiple gravity assist trajectories.

Therefore, numerous aspects of the multiple asteroid tour global optimization problem have been tackled in a number of ways by various authors, but a fully integrated, systematic approach has yet to be developed. Further advances in trajectory optimization methods and in computing power should soon enable the full global optimization problem to be solved.

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