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A LAGRANGIAN RELAXATION-BASED HEURISTIC APPROACH TO REGIONAL CONSTELLATION RECONFIGURATION PROBLEM

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A group of satellites—with either heterogeneous or homogeneous orbital characteristics and/or hardware specifications—can undertake a reconfiguration process due to variations in operations pertaining to regional coverage missions. This paper is concerned with the optimization of the specifications of the reconfiguration process that maximizes (resp., minimizes) the utility (resp., the cost). The specifications refer to the final design configuration and the transportation of satellites from one configuration to another. The utility refers to the coverage performance of the final configuration; the cost refers to the ΔV consumed or the time of flight incurred due to the reconfiguration process. We present an integer linear program formulation of the regional satellite constellation reconfiguration problem and two heuristic solution methods based on Lagrangian relaxation.

INTRODUCTION

Satellite constellation reconfiguration is defined as a process of transforming an existing configuration into a new configuration to maintain the system in an optimal state.[1,2] The potential factors that require a satellite constellation reconfiguration include: change in the mission coverage area, change in the number of satellites (addition[3] or loss[4]), and/or change in the coverage requirement.

The problem of reconfiguring satellite constellations generally consists of two independent problems—the constellation design problem and the constellation transfer problem.[2,5,6] The former is concerned with the design optimization of a new configuration that meets the emerging mission operational demand; the latter is concerned with the assignment of satellites from one configuration to another.[7] While the constellation reconfiguration problem may be approached in a design-in-series manner, i.e., design first and assignment last, the overall constellation reconfiguration process would be suboptimal due to lack of practical considerations such as the fuel state of satellites. For a few studies with concurrent consideration of these two aspects, the problems are often formulated as mixed-integer nonlinear programs that adopt meta-heuristic algorithms (e.g., a genetic algorithm) as optimization solvers.

The value of satellite constellation reconfiguration is amplified in the context of regional coverage missions. For example, a constellation monitoring disasters could be reconfigured to maximize the coverage over areas that are affected by natural disasters for increased bandwidth capacity. Prior research focused on the design of global coverage constellations such as the Streets-of-Coverage[8,9] or Walker patterns.[10–12] However, for regional coverage missions, the use of global coverage constellation methods would yield suboptimal coverage performance.[13]

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This work extends the prior work,[14] which introduced the mathematical formulation of the regional constellation reconfiguration problem (RCRP). Particularly, we improve the model of the problem for computational efficiency and present heuristic methods for RCRP.

BASIC CONCEPTS AND BACKGROUND

This section overviews the basic definitions and key properties of the regional coverage satellite constellation model by Lee et al.[13] For in-depth discussion on the theory and illustration of various examples, readers are encouraged to refer to Reference 13.

Model Assumptions

Two assumptions are made about the constellation model considering the mission context of interest: 1) the repeating ground track (RGT) orbits and 2) the common ground track constellation.

Assumption 1 (Repeating ground track orbit)**.** A satellite is placed on a repeating ground track orbit.

A ground track is the trace of satellite's sub-satellite points on the surface of a planetary body. A satellite on an RGT orbit makes N_{P} number of revolutions in N_{D} number of nodal periods. There is a finite time horizon of period T (often called a period of repetition) for which a satellite repeats its closed relative trajectory exactly and periodically. Expressing this condition:

$$T = N_{\mathrm{P}} T_{\mathrm{S}} = N_{\mathrm{D}} T_{\mathrm{G}}$$

where N_{P} and N_{D} are positive integers. T_{S} is the nodal period of a satellite due to both nominal motion and perturbations and T_{G} is the nodal period of Greenwich.

The rationale of the RGT orbit assumption for regional coverage is well supported in a study by Hanson et al.,[15] which has shown that RGT orbits yield better partial coverage performance than non-RGT orbits.

Assumption 2 (Common ground track constellation)**.** All satellites in a system follow a same relative trajectory.

All satellites in a common ground track constellation share identical semi-major axis a, eccentricity e, inclination i, and argument of periapsis ω but independently hold right ascension of ascending node (RAAN) Ω and mean anomaly M pairs that satisfy the following relation:[16]

$$N_{\mathrm{P}} \Omega_k + N_{\mathrm{D}} M_k = \text{constant mod } 2\pi$$

Considering the high orbital maintenance costs to negate perturbation effects, it is logical to assume only circular orbits or elliptic orbits with critical inclinations ($i \in \{63.4^\circ, 116.6^\circ\}$) for regional coverage missions.

Regional Coverage Satellite Constellation Model

In this constellation model, the finite time horizon of period T is discretized with time step size Δt. Let $\mathcal{T} \coloneqq \{0, 1, \ldots, m-1\}$ (where $|\mathcal{T}| = m = T/\Delta t$) be the set of time step indices t such that the set $\{t(\Delta t) : t \in \mathcal{T}\}$ is the discrete-time finite horizon. Let $\mathcal{J}$ be the set of orbital slot indices j along the common relative trajectory and $\mathcal{P}$ be the set of target point indices p.

Definition 1 (Reference visibility profile)**.** A reference satellite covers target point p if its elevation angle φ as viewed locally from p is greater than or equal to the minimum elevation angle threshold $\varphi_{\min}$ at time step t. The visibility indicator v_t at time step t is defined as follows:

$$v_t := \begin{cases} 1, & \text{if } \varphi \geq \varphi_{\min} \text{ at time step } t \\ 0, & \text{otherwise} \end{cases}$$

Then, a reference visibility profile $\boldsymbol{v}$ is simply an m-dimensional vector $\boldsymbol{v} = (v_t \in \{0, 1\} : t \in \mathcal{T})$.

To construct a reference visibility profile, the following parameters need to be specified: the orbital elements of a reference satellite $\text{\ae}_0 = (a, e, i, \omega, \Omega, M)$, the minimum elevation angle threshold $\varphi_{\min}$ that dictates the field-of-view of a satellite, the coordinates of a target point, and the epoch at which the finite time horizon is referenced to. With these parameters, the reference satellite is propagated under the governing equations of motion (e.g., J_2-perturbed Keplerian motion) for the finite time horizon of period T; at each time step t, the Boolean visibility mask ($\varphi \geq \varphi_{\min}$) is applied to construct an element of a reference visibility profile v_t.

Definition 2 (Visibility circulant matrix)**.** A visibility circulant matrix $\boldsymbol{V}$ is the $m \times m$ matrix whose columns are the cyclic permutations of $\boldsymbol{v}$:

$$\boldsymbol{V} = \begin{bmatrix} v_0 & v_{m-1} & \cdots & v_1 \\ v_1 & v_0 & \cdots & v_2 \\ \vdots & \vdots & \ddots & \vdots \\ v_{m-1} & v_{m-2} & \cdots & v_0 \end{bmatrix}$$

where the (t, j) entry of $\boldsymbol{V}$ is denoted with $V_{tj} = v_{(t-j) \bmod m}$.

Note that the first column of $\boldsymbol{V}$ is the reference visibility profile $\boldsymbol{v}$; this directly follows that the visibility circulant matrix is fully specified by a reference visibility profile.

Definition 3 (Constellation pattern vector)**.** A constellation pattern vector $\boldsymbol{x} = (x_j \in \{0, 1\} : j \in \mathcal{J})$ specifies the relative positioning of satellites along the common relative trajectory with respect to the reference satellite. Each element of $\boldsymbol{x}$ is defined as:

$$x_j := \begin{cases} 1, & \text{if a satellite occupies orbital slot } j \\ 0, & \text{otherwise} \end{cases}$$

where x_0 denotes the reference satellite position.

Definition 4 (Coverage timeline)**.** Let b_t be the number of satellite(s) in view from target point p at time step t. Then, we let $\boldsymbol{b} = (b_t \in \mathbb{Z}_+ : t \in \mathcal{T})$ denote a coverage timeline ($\mathbb{Z}_+$ denote the set of non-negative integers). Here, a visibility of a satellite from a target point follows from the Boolean elevation angle masking.

Remark 1 (Cyclic property). Under the aforementioned assumptions, a common RGT constellation system admits the *cyclic property*.[13] The cyclic property states that a visibility profile is simply a cyclic shift of the reference visibility profile. This follows from the fundamental assumptions of repeating ground track orbits and common ground track constellations.

Remark 2 (Circular convolution operation). Following from the cyclic property, we can relate reference visibility profile $\boldsymbol{v}$, constellation pattern vector $\boldsymbol{x}$, and coverage timeline $\boldsymbol{b}$ in the manner prescribed by a circular convolution operation. Mathematically,

$$b_t = \sum_{j \in \mathcal{J}} v_{(t-j) \bmod m} x_j \tag{1}$$

If $\mathcal{T} = \mathcal{J}$, then Eq. (1) can be written in terms of a reference visibility circulant matrix:

$$b_t = \sum_{j \in \mathcal{J}} V_{tj} x_j \tag{2}$$

which follows from the definition of the (t, j) entry of $\boldsymbol{V}$.

Example 1. We illustrate how all definitions of the model (so called the vectors of a system) come into play in a simple example. Let $\text{œ}_0 = (12\,758.5\,\text{km}, 0, 50°, 0°, 50°, 0°)$ (J2000) be the orbital elements of the reference satellite; this corresponds to the RGT ratio of $N_\text{P}/N_\text{D} = 6/1$, i.e., a satellite makes six revolutions in one nodal day. Assume a single target of interest p with the geodetic coordinate $(40°\text{N}, 100°\text{W})$. The minimum elevation angle threshold is set to $\varphi_\text{min} = 10\,\text{deg}$. Define an arbitrary constellation pattern vector $\boldsymbol{x}$ with length $m = 500$:

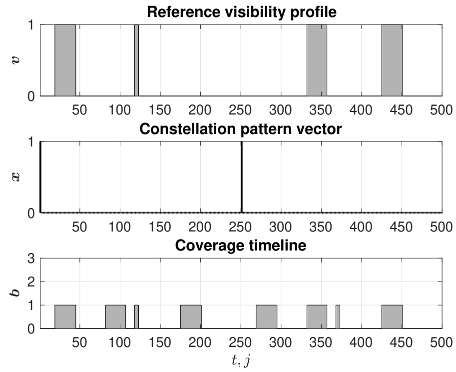
$$x_j = \begin{cases} 1, & \text{if } j \in \{0, 250\} \\ 0, & \text{otherwise} \end{cases}$$

When transcribed, this constellation pattern vector represents a constellation configuration consisting of two uniformly-distributed satellites along the common relative trajectory, each with its own unique orbital elements:

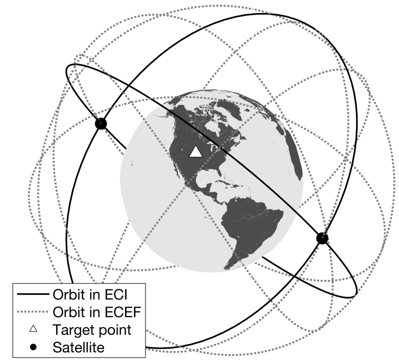
$$\begin{aligned} \text{œ}_1 &= (12\,758.5\,\text{km}, 0, 50°, 0°, 50°, 0°) \\ \text{œ}_2 &= (12\,758.5\,\text{km}, 0, 50°, 0°, 230°, 0°) \end{aligned}$$

The vectors of the system—$\boldsymbol{v}$, $\boldsymbol{x}$, and $\boldsymbol{b}$—are portrayed in Fig. 1a. The top part shows the reference visibility profile $\boldsymbol{v}$ which is constructed by propagating a hypothetical satellite with œ_0 for the finite time horizon of period T and applying the Boolean visibility mask at each time step. The constellation pattern vector $\boldsymbol{x}$ is shown in the middle. Note that in this example, $\text{œ}_0 = \text{œ}_1$ because $x_0 = 1$; this indicates that satellite 1 is essentially identical to the reference hypothetical satellite. The resulting coverage timeline $\boldsymbol{b}$, which follows directly from Eq. (1) of the cyclic property, for this two-satellite system is shown in the bottom part. The corresponding system is visualized in Fig. 1b in both Earth-centered inertial (ECI) and Earth-centered, Earth-fixed (ECEF) frames. Notice that there is a single closed trajectory in the ECEF frame (as opposed to two orbital planes in the ECI frame); each dot along the dotted curve represents an orbital slot.

In Example 1, we demonstrated the analysis of a known system using the vocabularies of the regional coverage satellite constellation model from Reference 13. In what follows next, we present a direct application of the model by formulating an integer linear programming problem with its decision variables being $\boldsymbol{x}$; instead of pre-specifying it, the goal is to find the optimal $\boldsymbol{x}^*$ that meets the specified coverage requirement.



(a) Illustration of $\boldsymbol{v}$, $\boldsymbol{x}$, and $\boldsymbol{b}$



(b) Corresponding constellation configuration in 3-D

Figure 1: Two-satellite system illustrated in Example 1

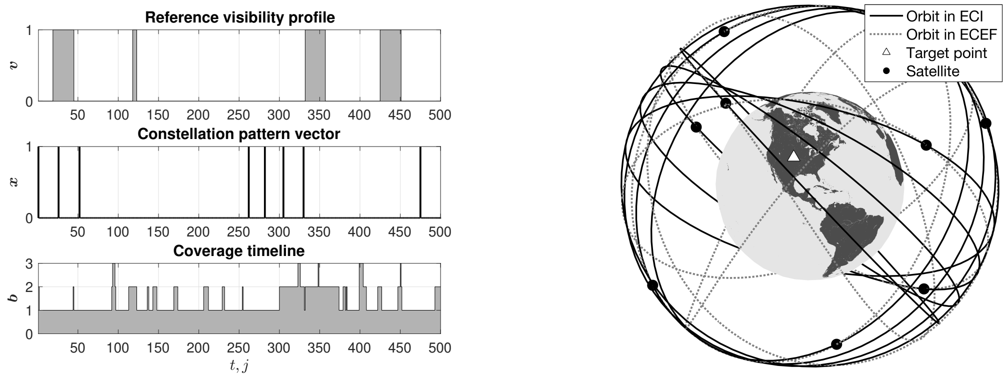
Model 1 (Regional constellation design problem)**.** Let $\mathcal{T} = \{0, \ldots, m-1\}$ be the set of time step indices and $\mathcal{J} = \{0, \ldots, m-1\}$ be the set of orbital slot indices. The regional constellation design problem (RCDP) is concerned with the optimization of the placement of satellites such that the number of satellites needed to satisfy the given coverage requirement $\boldsymbol{r} = (r_t \in \mathbb{Z}_+ : t \in \mathcal{T})$ imposed on a target point of interest p is minimized. The RCDP is formulated as an integer linear program:[13]

$$
\begin{aligned}
\text{(RCDP)} \quad \min \quad & \sum_{j \in \mathcal{J}} x_j \\
\text{s.t.} \quad & \sum_{j \in \mathcal{J}} V_{tj} x_j \geq r_t, \quad \forall t \in \mathcal{T} \\
& x_j \in \{0, 1\}, \quad \forall j \in \mathcal{J}
\end{aligned}
$$

where the decision variable $x_j = 1$ if a satellite occupies orbit slot j ($x_j = 0$ otherwise).

The problem is an instance of the general class of set covering problems whose goal is to minimize the cost (e.g., the number of sets) of covering all elements in the universe. Particularly, when the 0-1 integrality constraints are relaxed to the non-negative integrality constraints, the RCDP mimics the cyclic personnel staffing problem. The cyclic staffing problem is of significant interest to RCDP because it manifests that RCDP can be efficiently solved to optimality when the visibility profile contains a block of consecutive ones[17] (which is usually observable in cases with high-altitude orbits or low minimum elevation angle thresholds). When there are multiple blocks of consecutive ones, a guaranteed-accuracy heuristic approach is available[18] (e.g., a visibility profile depicted in Example 1 consists of four blocks of consecutive ones). Let us demonstrate RCDP in action.

Example 2. Consider the case illustrated in Example 1. Suppose we now want to provide a single-fold continuous coverage over the same target point p. This translates into $\boldsymbol{r} = \mathbb{1}$ where $\mathbb{1}$ is a vector of all ones. Solving the RCDP to optimality, we get the optimal constellation pattern vector $\boldsymbol{x}^*$ that consists of eight satellites (see the middle part of Fig. 2a). The corresponding optimal constellation configuration is pictured in Fig. 2b.



(a) Illustration of $\boldsymbol{v}$, $\boldsymbol{x}$, and $\boldsymbol{b}$

(b) Corresponding constellation configuration in 3-D

Figure 2: RCDP solution for Example 2

Constellation Transfer Problem

A constellation reconfiguration process incurs costs. In this subsection, we introduce an ILP formulation that models the constellation transfer aspect of the regional constellation reconfiguration problem. Note that the notations of decision variables and sets used in this model are only valid in the context of this problem.

Model 2 (Assignment problem)**.** Let $\mathcal{I} = \{1, \dots, n\}$ be the set of workers and $\mathcal{J} = \{1, \dots, m\}$ be the set of projects. The cost of assigning worker i to project j is denoted with c_{ij}. In the case of the unbalanced (i.e., when $n < m$) assignment problem (AP), the goal is to find the minimum cost assignment of n workers to m projects such that all workers are assigned to projects, but not all projects are assigned with workers. The AP can be formulated as an integer linear program:

$$
\begin{aligned}
\text{(AP)} \quad \min \quad & \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{j \in \mathcal{J}} x_{ij} = 1, \quad \forall i \in \mathcal{I} \\
& \sum_{i \in \mathcal{I}} x_{ij} \leq 1, \quad \forall j \in \mathcal{J} \\
& x_{ij} \in \{0, 1\}, \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}
\end{aligned}
$$

where the decision variable $x_{ij} = 1$ if worker i is assigned to project j ($x_{ij} = 0$ otherwise).

One special feature of AP is that the coefficient matrix of the constraints is totally unimodular and the right-hand sides are integral; hence, the problem can be efficiently solved as a linear program (e.g., the simplex algorithm) by relaxing the integrality constraints (i.e., the LP relaxation) and yet obtain integral optimal solutions. Many other specialized polynomial-time algorithms are also available such as the Hungarian algorithm[19] or the auction algorithm.[20]

The assignment problem received great attention in the field of satellite constellation reconfiguration research as an optimization model for a constellation transfer problem. Introduced in a study by de Weck et al.,[7] a constellation transfer problem may intuitively be modeled as AP using the following analogy: the satellites as the workers and the orbital slots as the projects; the coefficient c_{ij} is the cost of (e.g., the ΔV required or the time of flight) transferring satellite i to orbital slot j. The goal is to find the minimum ΔV assignment of n satellites to m orbital slots. In this paper, we adopt this transcription of the AP formulation in the modeling of a constellation transfer problem.

Constellation Design Problem

In this subsection, we focus on an ILP formulation that models the constellation design aspect of the regional constellation reconfiguration problem.

Model 3 (Maximum coverage problem)**.** Let $\mathcal{T} = \{0, \dots, m-1\}$ be the set of time step indices and $\mathcal{J} = \{0, \dots, m-1\}$ be the set of orbital slot indices. The goal of the maximum coverage problem (MCP) is to locate n satellites along the common relative trajectory such that the coverage over an area of interest is maximized. The MCP is formulated as an integer linear program:[14]

$$\text{(MCP)} \quad Z = \max \quad \sum_{t \in \mathcal{T}} w_t y_t \tag{3}$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{J}} V_{tj} x_j \geq r_t - M_t(1 - y_t), \quad \forall t \in \mathcal{T} \tag{4}$$

$$\sum_{j \in \mathcal{J}} x_j = n \tag{5}$$

$$x_j \in \{0, 1\}, \quad \forall j \in \mathcal{J}$$

$$y_t \in \{0, 1\}, \quad \forall t \in \mathcal{T}$$

where M_t are the big-M constants, and Z denote the optimal value of MCP. Constraint (5) is the cardinality constraint that restricts the number of satellites to n. In the objective function (3), w_t is the weight factor for each y_t. For simplicity, we let $w_t = 1, \forall t \in \mathcal{T}$ hereafter indicating equal-weight demand for coverage. The decision variable $x_j = 1$ if a satellite occupies orbital slot j ($x_j = 0$ otherwise) and

$$y_t = \begin{cases} 1, & \text{if the target is covered at time step } t \ (b_t \geq r_t) \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

We first note the difference in the formulations of two constellation design problems—RCDP and MCP. In the RCDP formulation, the number of satellites in a system is not restricted; hence, RCDP captures the concept of design optimization of an initial constellation configuration for complex regional coverage. The MCP formulation restricts the number of satellites in a system to a fixed value as in Constraint (5); this aligns with the notion of satellite constellation reconfiguration with a pre-specified number of active on-orbit satellites.

In the MCP formulation, the logical implications defined in Eq. (6) are linearized using the big-M formulation as shown in Constraints (4). It is evident that if $y_t = 0$, then Constraint (4) is satisfied provided that $M_t \geq \sup\{r_t - \sum_{j \in \mathcal{J}} V_{tj} x_j : t \in \mathcal{T}\}$. Because $\sum_{j \in \mathcal{J}} V_{tj} x_j = b_t \in \mathbb{Z}_+$ by Eq. (2), the condition simply becomes $M_t \geq r_t$.

The value of a big-M constant plays a critical role in an optimization process. To achieve the tightest LP relaxation bound with respect to the big-M constants, the tightest big-M constants are desirable for all constraints. The LP relaxation bound, which is the upper bound to the original

ILP, for MCP is $Z_{\text{LP}} = \sum_{t \in \mathcal{T}} y_t = 1 + \{\sum_{j \in \mathcal{J}}(V_{tj}x_j) - r_t\}/M_t$ assuming $w_t = 1, \forall t \in \mathcal{T}$. The last equality follows from the fact that MCP is a maximization problem—y_t variables will take their maximum values as bounded by Constraints (4). Expanding it further, we get the following analytical expression for Z_{LP}:

$$Z_{\text{LP}} = m - \sum_{t=1}^{m} \frac{r_t}{M_t} + \sum_{t=1}^{m} \frac{1}{M_t} \sum_{j=1}^{m} V_{tj} x_j \tag{7a}$$

$$= m - \sum_{t=1}^{m} \frac{r_t}{M_t} + n \sum_{t=1}^{m} \frac{V_{t1}}{M_t} + \sum_{j=2}^{m} x_j \Big\{ \sum_{t=1}^{m} \frac{V_{tj}}{M_t} - \sum_{t=1}^{m} \frac{V_{t1}}{M_t} \Big\} \tag{7b}$$

Consider Eq. (7a). It is evident that M_t needs to be minimized to tighten up the upper bound Z_{LP} and this occurs when $M_t = r_t$. Hence, we let $M_t = r_t, \forall t \in \mathcal{T}$ hereafter for the formulation of MCP such that the LP relaxation bound is the tightest; Constraints (4) then become:

$$\sum_{j \in \mathcal{J}} V_{tj} x_j \geq r_t y_t, \quad \forall t \in \mathcal{T} \tag{8}$$

Note that if $r_t = 1, \forall t \in \mathcal{T}$, the MCP with Constraints (8) reduces to the maximal covering location problem (MCLP) that emerges in many problem contexts (e.g., facility location problem). MCLP seeks to locate a number of facilities such that the coverage of demand nodes is maximized; each facility is pre-specified with a radius to which it can provide coverage. We have the following analogy: the satellites as the facilities and the time steps as the demand nodes. Unfortunately, this reduction informs us that MCP is NP-hard because of the NP-hardness of MCLP.[21] For more information on the mathematical formulation and the applications of MCLP, readers are encouraged to refer to the study by Church and ReVelle.[22]

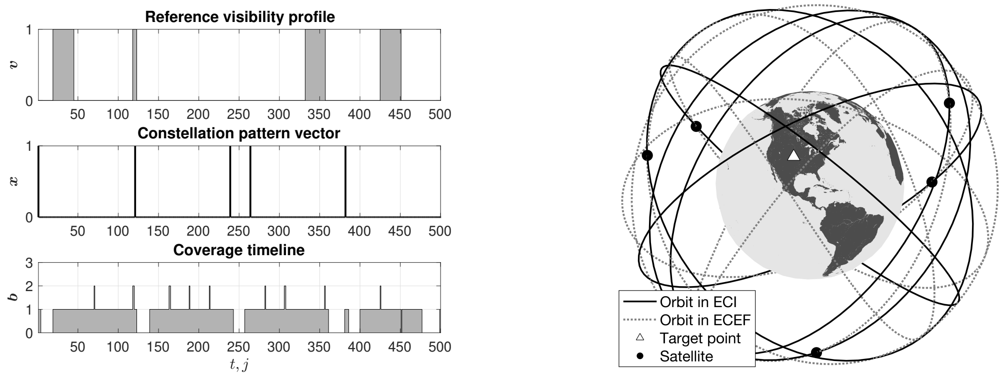
Example 3. Consider the case illustrated in Example 1. Suppose we now want to maximize the coverage over the same target point p with only five satellites. For the parameters, we let $\boldsymbol{c} = \mathbb{1}$ and $\boldsymbol{r} = \mathbb{1}$. Here, $\boldsymbol{r}$ represents the desired coverage state of target point p instead of the strict coverage requirement as defined in RCDP. Letting $\boldsymbol{w} = \mathbb{1}$ indicates that there is no preference on the time of day for coverage. Solving the MCP to optimality, we get the optimum of $Z^* = 398$, which translates into 79.6% temporal coverage of target point p by the optimized five-satellite configuration during the given repeat period T. The results are visualized in Fig. 3.

A constellation design problem can be modeled in any form possible with respect to many different coverage-related figures of merit (e.g., average revisit time, maximum revisit time) and not necessarily be restricted to the present MCP formulation. However, many of these metrics including the percent coverage share the notion of coverage maximization such that the minimization (or the maximization) of such respective metrics would necessarily maximize (or increase) the percent coverage metric. Therefore, considering the problem context herein, the percent coverage would suit the purpose as the representative figure of merit for the constellation design problem. Consideration of problem instances with different metrics is left for future work.

REGIONAL CONSTELLATION RECONFIGURATION PROBLEM

Problem Description

Suppose a group of satellites—with either heterogeneous or homogeneous orbital characteristics and/or hardware specifications—undertakes a reconfiguration process from configuration A to con-



(a) Illustration of $\boldsymbol{v}$, $\boldsymbol{x}$, and $\boldsymbol{b}$

(b) Corresponding constellation configuration in 3-D

Figure 3: MCP solution for Example 3

figuration B, of which the latter is undetermined. The problem is to find the specifications of the reconfiguration process that maximizes (resp., minimizes) the utility (resp., the cost) subject to some combination of variations in mission operations. Here, the specifications refer to both the design of configuration B and the transportation of satellites from one configuration to another.

The following mission operational decisions and external perturbations that give rise to a satellite constellation reconfiguration are captured in this paper. The subscripts A and B represent initial and final configuration, respectively.

1. Change in the mission coverage requirement, $\boldsymbol{r}_{\mathrm{A}} \to \boldsymbol{r}_{\mathrm{B}}$
   - Example coverage types: continuous coverage (single-fold, double-fold, etc.), intermittent (discontinuous) coverage, time-dependent coverage, etc.
   - Change in minimum elevation angle threshold, $\varphi_{\mathrm{min,A}} \to \varphi_{\mathrm{min,B}}$
2. Change in the area of interest, $\mathcal{P}_{\mathrm{A}} \to \mathcal{P}_{\mathrm{B}}$
   - Any target that can be represented as a set of discrete points (e.g., a point, a line, an area, or the globe).
3. Change in the number of satellites, $n_{\mathrm{A}} \to n_{\mathrm{B}}$
   - Addition of new satellites (e.g., capacity expansion)
   - Loss or retirement of existing satellites (e.g., loss reduction)

The variables (i.e., the specifications) of a reconfiguration process are:

1. Orbital characteristics of configuration B, $\boldsymbol{\text{œ}}_{0,\mathrm{B}}$
2. Distribution of satellites in configuration B, $\boldsymbol{x}_{\mathrm{B}}$

The figures of merit of a reconfiguration process are (including but not limited to):

1. Coverage timeline of configuration B, $\boldsymbol{b}_{\mathrm{B}}$
2. Costs such as the ΔV consumption or the time of flight

Mathematical Formulation

In this subsection, we propose a mathematical formulation of the regional constellation reconfiguration problem. The concept of *subconstellation* is applied.[13] We define following sets, parameters, and decision variables:

Sets and indices

$\mathcal{T}$	Set of time step indices (index t)
$\mathcal{I}$	Set of satellite indices (index i)
$\mathcal{S}$	Set of subconstellation indices (index s)
$\mathcal{J}_s$	Set of orbital slot indices of subconstellation $s \in \mathcal{S}$ (index j)
$\mathcal{P}$	Set of target point indices (index p)

Parameters

c_{ijs}	Cost of assigning satellite i to orbital slot j of subconstellation s
v_{tjps}	$\begin{cases} 1, & \text{if orbital slot } j \text{ of subconstellation } s \text{ is visible from target point } p \text{ at time step } t \\ 0, & \text{otherwise} \end{cases}$
r_{tp}	Coverage requirement of target point p at time step t

The mathematical formulation of the RCRP is as follows:

$$\text{(RCRP)} \quad \min \quad \sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} \sum_{i\in\mathcal{I}} c_{ijs} x_{ijs} \tag{9a}$$

$$\max \quad \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} y_{tp} \tag{9b}$$

$$\text{s.t.} \quad \sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} x_{ijs} = 1, \quad \forall i \in \mathcal{I} \tag{9c}$$

$$\sum_{i\in\mathcal{I}} x_{ijs} \leq 1, \quad \forall j \in \mathcal{J}_s, \forall s \in \mathcal{S} \tag{9d}$$

$$\sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} \sum_{i\in\mathcal{I}} v_{tjps} x_{ijs} \geq r_{tp} y_{tp}, \quad \forall t \in \mathcal{T}, \forall p \in \mathcal{P} \tag{9e}$$

$$x_{ijs} \in \{0, 1\}, \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}_s, \forall s \in \mathcal{S} \tag{9f}$$

$$y_{tp} \in \{0, 1\}, \quad \forall t \in \mathcal{T}, \forall p \in \mathcal{P} \tag{9g}$$

where the decision variables are

$$x_{ijs} = \begin{cases} 1, & \text{if satellite } i \text{ is allocated to orbital slot } j \text{ of subconstellation } s \\ 0, & \text{otherwise} \end{cases}$$

$$y_{tp} = \begin{cases} 1, & \text{if target point } p \text{ is covered at time step } t \ (b_{tp} \geq r_{tp}) \\ 0, & \text{otherwise} \end{cases}$$

The RCRP is formulated as a bi-objective integer linear program. The objective function (9a) minimizes the total cost (i.e., the total ΔV consumption or the total time of flight) of a constellation reconfiguration process; the objective function (9b) maximizes the temporal coverage over a set of target points. Constraints (9c) and (9d) are the AP-related constraints; Constraints (9c) ensure that every satellite is assigned to an orbital slot and Constraints (9d) restrict at most one satellite is occupied per orbital slot. Constraints (9e) are the MCP-related constraints; these constraints ensure that the target point p is covered at time step t only if there exists at least r_{tp} satellite(s) in view. Note that the cardinality constraint [Constraint (5)] of MCP is omitted because it is implied by the satellite indices set $\mathcal{I} = \{1, \ldots, n\}$ and the AP-related constraints. Constraints (9f) and (9g) define the domains of decision variables.

Notice the decision variables of RCRP—they are in the form of the AP decision variables; the reasoning behind this choice is explained. The decision variable $x_{ijs}^{(\mathrm{AP})}$ of AP indicates an assignment of satellite i to orbital slot j of subconstellation s while the decision variable $x_{js}^{(\mathrm{MCP})}$ of MCP indicates whether a satellite occupies orbital slot j of subconstellation s. Therefore, it follows naturally that $x_{ijs}^{(\mathrm{AP})}$ are the elemental decision variables (see Fig. 4) because $x_{js}^{(\mathrm{MCP})}$ can be deduced from $x_{ijs}^{(\mathrm{AP})}$. These two different sets of decision variables are coupled through the following relationship along with Constraints (9c) and (9d):

$$x_{js}^{(\mathrm{MCP})} = \sum_{i \in \mathcal{I}} x_{ijs}^{(\mathrm{AP})}, \quad \forall j \in \mathcal{J}, \forall s \in \mathcal{S} \tag{10}$$

where both $x_{ijs}^{(\mathrm{AP})}$ and $x_{js}^{(\mathrm{MCP})}$ are binary variables.

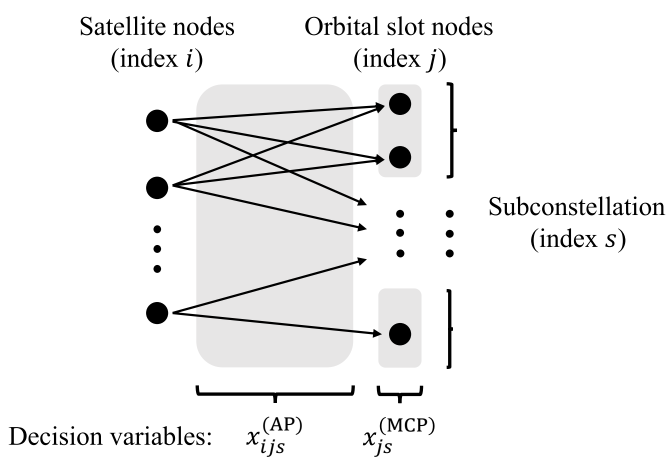


Figure 4: Decision variables of AP and MCP and their relationship

This coupled relationship in Eq. (10) enables an integrated ILP formulation that simultaneously considers both the constellation transfer problem and the constellation design problem; both the regional coverage and the n-fixed formulation aspects are embedded in this relationship.

The initial configuration need not necessarily be a common RGT constellation but can be a group of satellites in any orbit (e.g., non-RGT orbits, elliptic, circular, etc.) and/or with different sensor specifications. This is because the model is concerned with the assignment of a satellite to an orbital slot, not the bipartite matching between the two sets of common RGT orbital slots. Hence, the model is suitable for accounting both the regional-to-regional and non-regional-to-regional reconfigurations.

Model Characteristics

Remark 3. The AP structure is preserved in RCRP with the decision variables being AP-based. In this perspective, the RCRP is a bi-objective AP with complicating logical constraints. The logical constraints are Constraints (9e).

Following the discussion from Remark 3, it is evident that whenever Constraints (9e) are inactive, the RCRP can be solved as a bi-objective AP. The RCRP is NP-hard whenever at least one of Constraints (9e) is active. This observation follows from the NP-hardness of MCLP,[21] which is shown to be a special case of MCP.

SOLUTION METHODS

This section discusses solution methods to the regional constellation reconfiguration problem. We propose two solution methods, 1) a Lagrangian relaxation-based heuristic approach based on the epsilon-constraint reformulation and 2) a Lagrangian relaxation-based heuristic solution approach based on the weighted-sum reformulation. Both methods employ the Lagrangian relaxation method. However, the major difference between these methods exists in the heuristic algorithm to find feasible solutions.

Lagrangian Heuristics based on Epsilon-constraint Reformulation

The goal of RCRP, as implied by its formulation, is to identify non-dominated solutions. The problem is reformulated as a single-objective optimization problem via the epsilon-constraint method such that the non-dominated solutions can be found by solving epsilon-constrained RCRP (ε-RCRP) sequentially. Algorithm 1 discusses the overall procedure.

Given a parameter ε, the epsilon-constrained single-objective model is formulated as:

$$\begin{aligned}
(\varepsilon\text{-RCRP}) \quad Z(\varepsilon) = \min \quad & \sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} \sum_{i\in\mathcal{I}} c_{ijs} x_{ijs} \\
\text{s.t.} \quad & \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} y_{tp} \geq \varepsilon \qquad (11) \\
& \text{Constraints (9c)–(9g)}
\end{aligned}$$

where $Z(\varepsilon)$ denote the optimal value of ε-RCRP.

In ε-RCRP, the objective function (9b) is transformed into a constraint that is bounded from below by the parameter ε as shown in Constraint (11).

Algorithm 1 Epsilon-constraint method

```
1: procedure EPSILON-CONSTRAINT METHOD
2:     Initialize ε ← ε_0
3:     while ε-RCRP is feasible do
4:         Z(ε) ← ε-RCRP
5:         ε ← ε + ε_step
6:     end while
7:     return List of Z(ε) values
8: end procedure
```

Lower Bound: Lagrangian Relaxation The Lagrangian relaxation is a computational technique to approach the difficult problem by dualizing complicating constraints in an original problem such that the remaining structure is efficiently solved. The Lagrangian relaxation provides a lower (resp., upper) bound to the original minimization (resp., maximization) problem. Specifically, in our case, the complicating constraints can be viewed as the MCP-related constraints [Constraints (9e)] primarily due to the intact AP structure in the relaxed problem (Remark 3).

To retrieve the *Lagrangian problem* (LR) of ε-RCRP, we dualize Constraint (9e):

$$
\begin{aligned}
\text{(LR)} \quad Z_{\mathrm{D}}(\varepsilon, \boldsymbol{\lambda}) = \min \quad & \sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} \sum_{i\in\mathcal{I}} c_{ijs} x_{ijs} - \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} \lambda_{tp} \left[\sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} \sum_{i\in\mathcal{I}} v_{tjps} x_{ijs} - r_{tp} y_{tp} \right] \\
\text{s.t.} \quad & \sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} x_{ijs} = 1, \quad \forall i \in \mathcal{I} \\
& \sum_{i\in\mathcal{I}} x_{ijs} \leq 1, \quad \forall j \in \mathcal{J}_s, s \in \mathcal{S} \\
& \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} y_{tp} \geq \varepsilon \\
& x_{ijs} \in \{0,1\}, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_s, s \in \mathcal{S} \\
& y_{tp} \in \{0,1\}, \quad \forall t \in \mathcal{T}, p \in \mathcal{P}
\end{aligned}
$$

where $\boldsymbol{\lambda} = (\lambda_{tp} \in \mathbb{R}_+ : t \in \mathcal{T}, p \in \mathcal{P})$ is a vector of Lagrange multipliers, and $Z_{\mathrm{D}}(\varepsilon, \boldsymbol{\lambda})$ denote the optimal value of LR.

The optimal value of LR can be tightened by solving for the optimal $\boldsymbol{\lambda}$. Such a problem is called the *Lagrangian dual problem* and is formulated as follows:

$$
Z_{\mathrm{D}}(\varepsilon) = \max_{\boldsymbol{\lambda}} Z_{\mathrm{D}}(\varepsilon, \boldsymbol{\lambda}) \tag{12}
$$

To solve the Lagrangian dual problem, we use the subgradient optimization method, which has been shown to be an effective method for various problem settings. For more information on the implementation of the subgradient optimization method, readers are encouraged to refer to Reference 23.

Remark 4. The Lagrangian problem of ε-RCRP can be decomposed into two subproblems.

$$
\begin{aligned}
\text{(LR1)} \quad Z_{\mathrm{D1}}(\boldsymbol{\lambda}) = \min \quad & \sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} \sum_{i\in\mathcal{I}} c_{ijs} x_{ijs} - \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} \lambda_{tp} \left[\sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} \sum_{i\in\mathcal{I}} v_{tjps} x_{ijs} \right] \\
\text{s.t.} \quad & \sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} x_{ijs} = 1, \quad \forall i \in \mathcal{I} \\
& \sum_{i\in\mathcal{I}} x_{ijs} \leq 1, \quad \forall j \in \mathcal{J}_s, s \in \mathcal{S} \\
& x_{ijs} \in \{0,1\}, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_s, s \in \mathcal{S}
\end{aligned}
$$

$$
\begin{aligned}
\text{(LR2)} \quad Z_{\mathrm{D2}}(\varepsilon, \boldsymbol{\lambda}) = \min \quad & \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} \lambda_{tp} r_{tp} y_{tp} \\
\text{s.t.} \quad & \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} y_{tp} \geq \varepsilon \\
& y_{tp} \in \{0,1\}, \quad \forall t \in \mathcal{T}, p \in \mathcal{P}
\end{aligned}
$$

Remark 5. The Lagrangian problem of ε-RCRP can be efficiently solved:

- The constraint coefficient matrix of LR1 is totally unimodular; solve LR1 via LP relaxation.
- Solve LR2 using conventional ILP.

Remark 6. The optimal value of the Lagrangian dual problem is a lower bound to the optimal value of ε-RCRP. Mathematically, the following relationship holds: $Z_{\mathrm{D}}(\varepsilon) \leq Z(\varepsilon)$.

Upper Bound: Constructing Feasible Solutions By solving the Lagrangian dual problem, we obtain a lower bound to ε-RCRP although its optimal solution may be infeasible to ε-RCRP. To produce feasible solutions, we exploit the behavior of the subgradient optimization. We observe that while solving the Lagrangian dual problem using the subgradient optimization, feasible solutions are often produced, if not almost feasible solutions in many cases. The idea is to drive these solutions to feasibility by employing a heuristic approach. Such an approach based on the Lagrangian relaxation is called the Lagrangian heuristics in the literature.

To construct feasible solutions from the subgradient optimization, we employ the following algorithm. First, we begin by initializing the parameters of the subgradient optimization method. These initial values are adopted from Reference 23, which have been shown to be effective empirically. Given $\boldsymbol{\lambda}$, ε, and other subgradient optimization parameters, we solve the Lagrangian problem of ε-RCRP in a decomposed manner. The value Z_{D} then would become a lower bound at that iteration.

To produce an upper bound at each iteration, the optimal solution $\boldsymbol{x}$ from LR1 is modified and taken as a parameter to ε-RCRP. The idea is to remove one or more non-zero elements from $\boldsymbol{x}$ and solve the reduced dimension ε-RCRP (Reduced ε-RCRP). The desired coverage state $\boldsymbol{r}$ should also be appropriately modified. If Reduced ε-RCRP is feasible, then we have found an upper bound feasible solution; otherwise, a different set of non-zero elements should be removed from $\boldsymbol{x}$. The algorithm re-iterates until a feasible solution is obtained.

The described process is an algorithm for a single iteration of the subgradient optimization method. On a high level, the subgradient optimization is performed to solve the Lagrangian dual problem. The Lagrange multipliers are updated according to the rule by Fisher.[23] The subgradient optimization is terminated if the maximum number of iterations is achieved or the gap between the lower bound and the upper bound satisfies the pre-defined gap tolerance. In this process, both the lower bound and the upper bound get updated with their best values. The best upper bound would then be the feasible solution to the ε-RCRP given ε as the parameter. The process mentioned here refers to Line 4 of Algorithm 1.

Lagrangian Heuristics based on Weighted Sum Reformulation

Another Lagrangian heuristic approach is developed based on the weighted sum reformulation of RCRP. The weighted sum is a widely used method in approaching the multi-objective optimization problem along with the epsilon-constraint method. As will be discussed later in this subsection, there is a distinct feature associated with this approach when constructing feasible solutions.

The application of the Lagrangian relaxation is similar to that of the epsilon-constraint method. We first apply the weighted sum scheme on RCRP (WS-RCRP):

$$\text{(WS-RCRP)} \quad Z(\mu) = \min \quad \mu \sum_{s \in \mathcal{S}} \sum_{j \in \mathcal{J}_s} \sum_{i \in \mathcal{I}} c_{ijs} x_{ijs} - (1-\mu) \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} y_{tp}$$
$$\text{s.t.} \quad \text{Constraints (9c)–(9g)}$$

where $\mu \in [0,1]$ is the relative weight factor, and $Z(\mu)$ denote the optimal value of WS-RCRP.

To identify the Pareto efficient solutions using the weighted sum approach, one must enumerate each WS-RCRP to optimality while varying μ; this concept is similar to that of the epsilon-constraint method.

Due to the complexity of the problem, the Lagrangian relaxation of the complicating constraints is applied in each WS-RCRP. The Lagrangian problem is obtained by dualizing Constraints (9e):

$$\begin{aligned}
\text{(LR)} \quad Z_{\text{D}}(\mu, \boldsymbol{\lambda}) = \min \quad & \mu \sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} \sum_{i\in\mathcal{I}} c_{ijs} x_{ijs} + (\mu - 1) \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} y_{tp} \\
& - \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} \lambda_{tp} \left[\sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} \sum_{i\in\mathcal{I}} v_{tjps} x_{ijs} - r_{tp} y_{tp} \right] \\
\text{s.t.} \quad & \text{Constraints (9c),(9d),(9f),(9g)}
\end{aligned}$$

where we denote $Z_{\text{D}}(\mu, \boldsymbol{\lambda})$ the optimal value of LR.

Note that the application of the weighted sum scheme should occur after the application of the Lagrangian relaxation. Dualizing Constraints (9e) and applying the weighted sum scheme would yield incorrect solutions.

Similarly as before, we have the Lagrangian dual problem formulated as follows:

$$Z_{\text{D}}(\mu) = \max_{\boldsymbol{\lambda}} Z_{\text{D}}(\mu, \boldsymbol{\lambda}) \tag{13}$$

Remark 7. The Lagrangian problem of WS-RCRP can be decomposed into two subproblems.

$$\begin{aligned}
\text{(LR1)} \quad Z_{\text{D1}}(\mu, \boldsymbol{\lambda}) = \min \quad & \mu \sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} \sum_{i\in\mathcal{I}} c_{ijs} x_{ijs} - \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} \lambda_{tp} \left[\sum_{s\in\mathcal{S}} \sum_{j\in\mathcal{J}_s} \sum_{i\in\mathcal{I}} v_{tjps} x_{ijs} \right] \\
\text{s.t.} \quad & \text{Constraints (9c),(9d),(9f)}
\end{aligned}$$

$$\begin{aligned}
\text{(LR2)} \quad Z_{\text{D2}}(\mu, \boldsymbol{\lambda}) = \min \quad & (\mu - 1) \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} y_{tp} + \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} \lambda_{tp} r_{tp} y_{tp} \\
\text{s.t.} \quad & \text{Constraints (9g)}
\end{aligned}$$

Remark 8. The Lagrangian problem of WS-RCRP can be efficiently solved:

- The constraint coefficient matrix of LR1 is totally unimodular; solve LR1 via LP relaxation.
- Solve LR2 trivially. If coefficient i is negative, then corresponding $y_i = 1$ ($y_i = 0$ otherwise).

Remark 9. The optimal value of the Lagrangian dual problem is a lower bound to the optimal value of WS-RCRP. Mathematically, the following relationship holds: $Z_{\text{D}}(\mu) \leq Z(\mu)$.

Remark 10. The Lagrangian problem of WS-RCRP possesses the integrality property.[24] That is, $Z_{\text{LP}}(\mu) = Z_{\text{D}}(\mu)$.

Remark 10 simply states that the optimum obtained by solving the Lagrangian dual problem would be equal to that of the LP relaxation problem. Although the lower bound cannot be tighter than the LP relaxation bound, the value of the heuristic approach lies in the application of the subgradient optimization.

There is an apparent benefit associated with the weighted sum approach. The heuristic to find feasible solutions is straightforward—the subgradient optimization always produces feasible upper bound solutions. This is because there is no epsilon-constraint imposed on $\boldsymbol{y}$ that causes inconsistency between $\boldsymbol{x}$ and $\boldsymbol{y}$ optimal solutions.

COMPUTATIONAL STUDY

For the purpose of demonstrating the solution methods, we adopt the case study introduced in Reference 14.

Experimental Design

Let us suppose a constellation system consisting of five satellites. These satellites are categorized into two unique subconstellations. We define reference satellite orbital elements for each subconstellation: for subconstellation 1, let $\text{œ}_0 = (10\,527.4\,\text{km}, 0, 70°, 0°, 0°, 0°)$; for subconstellation 2, let $\text{œ}_0 = (12\,758.4\,\text{km}, 0, 47.92°, 0°, 0°, 0°)$. Three satellites are members of subconstellation 1 whereas two satellites are members of subconstellation 2. In this example, we let $m = 500$ per subconstellation. Following are the initial constellation pattern vectors:

$$x_{j1} = \begin{cases} 1, & \text{for } j = 67, 155, 285 \\ 0, & \text{otherwise} \end{cases}$$

$$x_{j2} = \begin{cases} 1, & \text{for } j = 199, 399 \\ 0, & \text{otherwise} \end{cases}$$

The goal is to reconfigure the existing five-satellite system along with two new satellites (i.e., $n = 7$) so as to increase coverage over new target points of interest: $p_1 = (34.1°\text{N}, 118.5°\text{W})$ and $p_2 = (12.9°\text{N}, 12.0°\text{E})$. These new satellites are assumed to be deployed simultaneously by a carrier vehicle at position x_{01}. For both target points, we let $\boldsymbol{r} = \mathbb{1}$ and $\varphi_{\min} = 10\,\text{deg}$. Note that both target points have equal-weight demand for coverage.

Results and Discussion

The problem is approached in three ways. In order to gauge the performance of the heuristic solution, we first use a conventional mixed integer programming (MIP) solver, Gurobi 9.0 solver, to approximate the true Pareto frontier by solving multiple ε-RCRP sequentially. During this process, we relax the integrality constraints and obtain the LP relaxation lower bound. We used the developed Lagrangian heuristic methods based on the epsilon-constraint reformulation and the weighted sum reformulation.

The results are summarized and visualized in Figure 5 and Table 1. First, we focus on the leftmost solution. The original five-satellite system with two additional satellites provide 62.4% and 56.8% coverage over p_1 and p_2, respectively. The total ΔV consumed is 0.11 km/s. This is, in fact, the minimal-fuel-consumption strategy, and the only consumption of the fuel is due to the fact that no two satellites are allowed to occupy the same orbital slot [see Constraints (9e)]; in this case, one of the two new satellites performed the maneuver to satisfy this constraint. We refer to this solution as the AP threshold because the same result can be obtained by solving the AP subproblem.

Overall, both the epsilon-constraint Lagrangian heuristic and the weighted sum Lagrangian heuristic performed well in low-percent coverage instances. The epsilon-constraint Lagrangian heuristic becomes computationally intractable at high-percent coverage instances, primarily because multiple Reduced ε-RCRPs are solved sequentially using the conventional MIP solver and the dimension of Reduced ε-RCRP becomes larger as more satellites are required to relocate to satisfy Constraint (11).

The weighted sum Lagrangian heuristic scored the least mean computation time as shown in Table 1 because the subgradient optimization solutions are always feasible. However, the weighted sum Lagrangian heuristic needed to call much more instances of WS-RCRP to improve the quality of the approximated Pareto frontier; out of 1001 instances, 67 solutions are on the approximated Pareto frontier. As shown in Figure 6, a larger number of WS-RCRP instances do lead to a better Pareto frontier approximation; however, this comes at the cost of additional computational cost.

It is worth mentioning that the gap between the true Pareto efficient solutions and the LP relaxation solutions is significant at high-percent coverage instances. Hence, for the weighted sum Lagrangian heuristic method, which possesses the integrality property, the metric such as the integrality gap alone would not provide any meaningful implications.

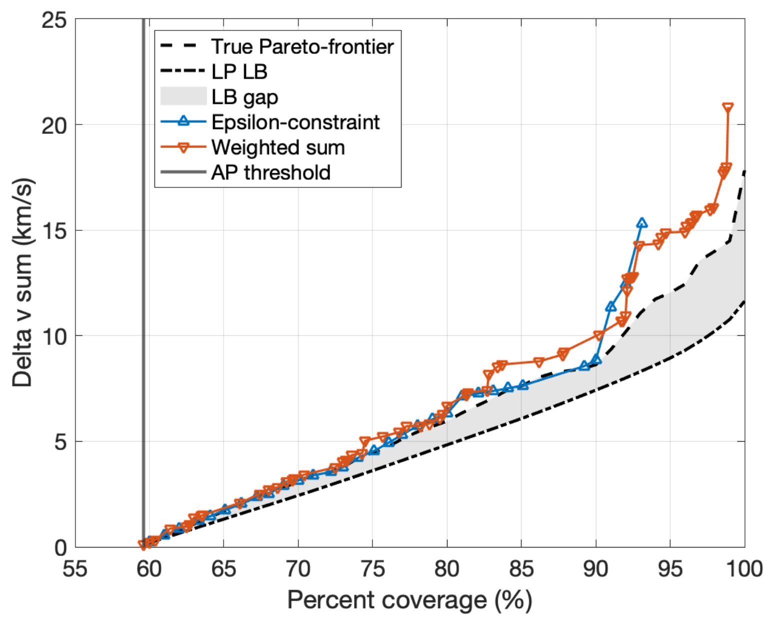


Figure 5: Approximated Pareto frontiers of the solution methods

CONCLUSION

In this paper, we extended the prior work on the regional constellation reconfiguration problem by improving the MCP model and proposing the bi-objective formulation of RCRP. To solve the problem efficiently, we proposed two heuristic solution methods based on the Lagrangian relaxation technique. The numerical analysis shows that the epsilon-constraint-based Lagrangian heuristic method is computationally efficient for low-percent coverage cases primarily due to the fact that Reduced ε-RCRP is effective for small-dimension instances. In contrast, the weighted sum-based

Table 1: Comparison of solution methods

Method	Computation time (s)				No. of ε,μ iter.	No. of Pareto sol.
	Sum	Mean	Min.	Max		
MIP	76403.59	1863.50	2.44	20207.79	41	41
ε-constraint LH	5894.61	173.37	0.73	1174.34	34	31
Weighted sum LH	11100.68	11.08	7.51	29.01	1001	67
	23248.44	11.62	7.44	29.33	2001	78

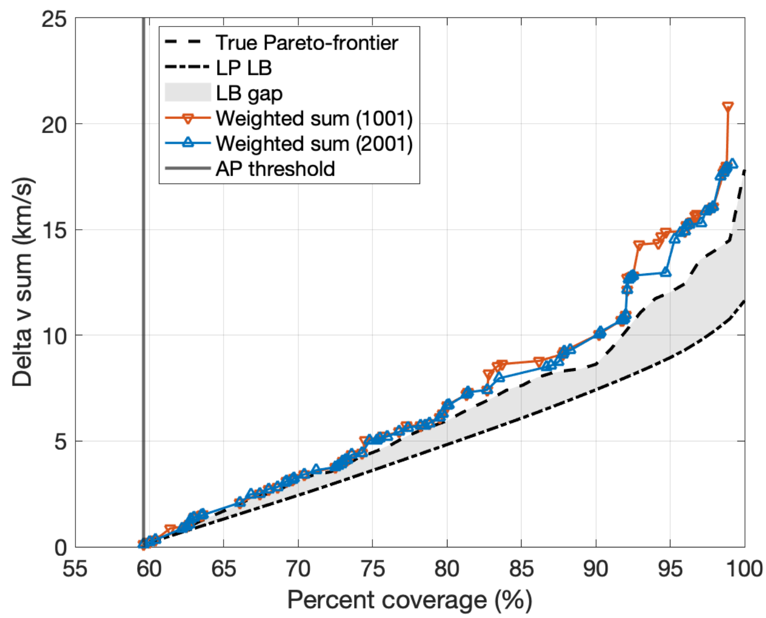


Figure 6: Comparison of weighted sum Lagrangian heuristics with different number of WS-RCRP instances

Lagrangian heuristic method is computationally light at each iteration of the subgradient optimization but requires a large number of WS-RCRP iterations to approximate the Pareto frontier. Overall, the analysis of the results shows that both heuristic methods possess the potential to be faster alternatives to the conventional MIP solver at the cost of solution quality.

Several directions for future work are discussed. The weighted sum-based Lagrangian heuristic method retains inherent disadvantages of the weighted sum method such as neglecting optimal solutions in non-convex regions. Several advanced weighted sum techniques (e.g., the adaptive weighted sum method[25]) could potentially improve the overall process of approximating the Pareto front. The current implementation of the epsilon-constraint-based Lagrangian heuristic method focuses on reducing the dimension of the problem with respect to the number of satellites. Considering the fact that for most instances of the problem, $|\mathcal{I}| < |\mathcal{J}|$, and in order to further reduce the dimension of the problem, one could implement a pruning heuristic to eliminate orbital slots of less significance. Such an approach could lead to faster overall computation time.

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