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Response Surface Equations for Expendable Launch Vehicle Payload Capability

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Systems analysis and conceptual design for new spacecraft commonly require the capability to perform rapid, parametric assessments of launch vehicle options. Such assessments allow engineers to incorporate launch vehicle considerations in first-order cost, mass, and orbit performance trades early during conceptual design and development phases. This paper demonstrates an efficient approach to launch vehicle analysis and selection using response surface equations (RSEs) derived directly from launch vehicle payload planner's guides. These RSEs model payload capability as a function of circular orbit altitude and inclination. Following presentation of the RSE fitting method and statistical goodness of fit tests, the RSE and model fit error statistics for the Pegasus XL are derived and presented as an example. In total, 43 RSEs are derived for the following launch vehicles and their derivatives: Pegasus, Taurus, Minotaur, and Falcon series as well as the Delta IV, Atlas V, and the foreign Ariane and Soyuz vehicles. Ranges of validity and model fit error statistics with respect to the original planner's guide data are provided for each of the 43 fits. Across all launch vehicles fit, the resulting RSEs have a maximum 90th percentile model fit error of 4.39% and a mean 90th percentile model fit error of 0.97%. In addition, of the 43 RSEs, the lowest R^2 value is 0.9715 and the mean is 0.9961. As a result, these equations are sufficiently accurate and well-suited for use in conceptual design trades. Examples of such trades are provided, including demonstrations using the RSEs to (1) select a launch vehicle given an orbit inclination and altitude, (2) visualize orbit altitude and inclination constraints given a spacecraft mass, and (3) calculate the sensitivity of orbital parameters to mass growth. Suited for a variety of applications, the set of RSEs provides a tool to the aerospace engineer allowing efficient, informed launch option trades and decisions early during design.

Nomenclature

ELV	=	Expendable Launch Vehicle
h	=	circular orbit altitude, km
HAPS	=	Hydrazine Auxiliary Propulsion System
i	=	circular orbit inclination, deg.
MFE	=	Model Fit Error
m_{pay}	=	launch vehicle payload capability, kg
RSE	=	Response Surface Equation
RSM	=	Response Surface Methodology
x_k	=	predictor variable
y	=	response variable
β_k	=	partial regression coefficient

I. Introduction

A common requirement in systems analysis and conceptual design for new spacecraft is the capability to perform rapid, parametric assessments of launch vehicle options. Such assessments allow engineers to incorporate launch vehicle considerations in first-order cost, mass, and orbit performance trades early during conceptual design and development phases. Often, such launch vehicle analysis is accomplished through manual references to sources such as launch-vehicle-specific payload planner's guides. This method can be time consuming and is not conducive to parametric exploration and trade studies. In this paper, we describe a response surface fitting method and derive response surface equations describing payload capability for a large set of expendable launch vehicles (ELVs) in order to enable more efficient launch option analyses for a variety of space applications.

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Response surface methodology (RSM) provides parametric mathematical models of data series by approximating underlying dependencies between inputs and outputs using a polynomial relationship. The resulting polynomial model is known as a response surface equation (RSE). For a launch vehicle delivering a payload to a circular Earth orbit, the maximum payload capability depends on two main independent variables: orbit altitude and inclination. Thus, in the response surface equations documented here, the inputs are altitude and inclination and the output (response) is payload capability to orbit.

In this work, we derive and analyze 43 RSEs for the following launch vehicles and their derivatives: Pegasus, Taurus, Minotaur, and Falcon series as well as the Delta IV, Atlas V, and the foreign Ariane and Soyuz vehicles. The results here provided should be useful to spacecraft systems engineers and mission planners in allowing integrated, extensive, and efficient launch options analyses and parametric trade studies (of cost and payload mass to orbit, for example) early during conceptual design and development phases. An illustrative launch scenario of the 2006 TacSat-2 spacecraft is used to demonstrate several different applications of these RSEs in answering common satellite design questions. These questions include how to (1) select a launch vehicle given an orbit inclination and altitude, (2) visualize orbit constraints and trades given a spacecraft mass, and (3) calculate sensitivity of orbital parameters to mass growth.

II. Response Surface Fitting Method

To analyze the maximum payload capability of a launch vehicle, mission planners and engineers frequently refer to plots or tables in the vehicle's payload planner's guide. This method can be time consuming, and it is limited in its ability to quickly and efficiently analyze multiple vehicles and/or orbits. Figure 1 shows the typical layout for a plot in a payload planner's guide for the air-launched Pegasus XL.

As Fig. 1 shows, if a mission planner desires, for example, to launch a payload to an orbit altitude of 500 km and inclination of 45°, the Pegasus XL can carry a maximum payload of approximately 350 kg. If the mission planner wishes to examine another possible mission profile with a desired altitude of 750 km at a 45° inclination, the figure look-up process shows that the maximum payload for the Pegasus is reduced to approximately 300 kg. If a planner wishes to compare vehicles, this process must be repeated for the alternative vehicles. This trade exploration process becomes simpler with a polynomial regression, or response surface equation, representation of these plots.

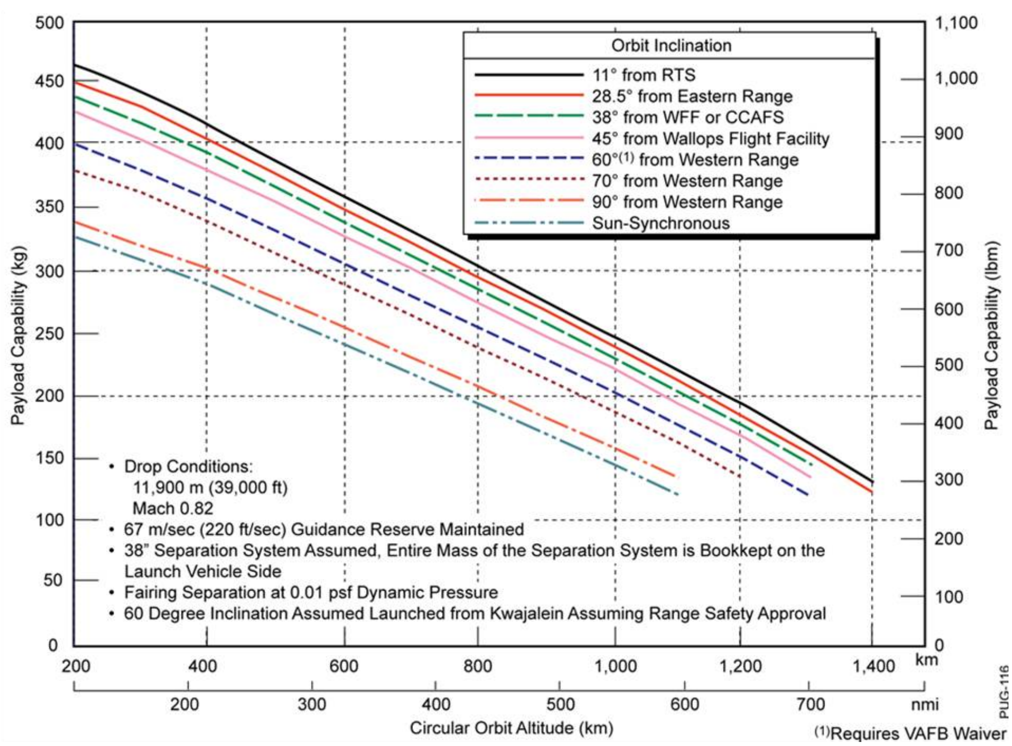

Figure 1. Pegasus XL Performance Capability.[1]

A. Introduction to Response Surface Methodology

Given a set of data points, response surface methodology (RSM) establishes analytical relationship between several independent variables and one (or more) dependent variables. RSM can provide significant insight to previously unknown or complicated response behavior. In a situation where several input variables (for this paper, altitude and inclination) potentially influence some performance measure (e.g., maximum payload capability), RSM provides a means to analyze this influence. By performing a least squares linear regression on the coefficients of a multivariate Taylor expansion, an RSE can be fit to any set of data. The general form of a first-order RSE is given in Eq. (1):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k \qquad (1)$$

Here, y represents the response variable, β_0 is the intercept, β_k the partial regression coefficients, and x_k the predictor variables or regressors.[2] For this work, a second-order response surface model is used as shown in Eq. (2):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2 \qquad (2)$$

In this paper, the dependent response variable, y in Eq. (2), is the maximum payload capability of a launch vehicle and will be noted hereafter as m_{pay}. The variable x_1 is orbit altitude (also called h) and x_2 is orbit inclination (also called i). Thus, for this application, Eq. (2) becomes:

$$m_{pay} = \beta_0 + \beta_1 h + \beta_2 i + \beta_{11} h^2 + \beta_{22} i^2 + \beta_{12} ih \qquad (3)$$

The work presented in Section III focuses on the determination of the partial regression coefficients for a set of launch vehicles and analysis of the quality of fit.

B. Response Surface Fitting Procedure

In order to determine the proper RSE partial regression coefficients for each launch vehicle, a standard fitting method is here employed and described next. For all domestic launch vehicles, the data to which the RSE is fit is compiled from publicly-available payload planner's guides.[1,3-9] The data for foreign ELVs is gathered from Ref. 10. In most cases, this data exists in the form of plots. Engauge Digitizer software[11] is used to convert the plot images into numerical tables of data points. Engauge automatically recognizes lines in the image and highlights line segments. Figure 2 shows a screen shot of the Engauge software as it converts Pegasus plot data into a table of payload capability as a function of circular orbit altitude for a given inclination; the process is repeated for each inclination provided by the planner's guide. The resulting three-column table provides payload capability as a function of circular orbit altitude and inclination for the vehicle.

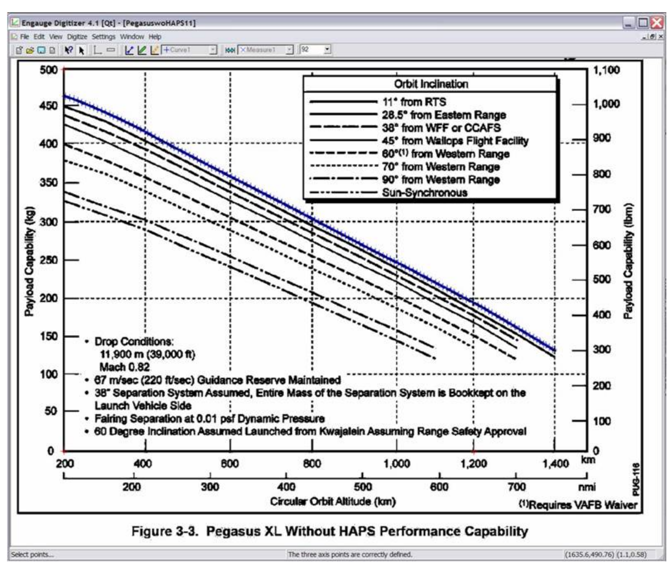

Figure 2. Engauge Digitizer interface during a Pegasus XL figure-to-table conversion.

The table from the Engauge digitization is then imported into the JMP statistical software package for RSE least-squares regression fitting.[12] For a given launch vehicle, JMP outputs a set of partial regression coefficients (β values) corresponding to the second-order model in Eq. (3). The goodness of fit is verified through a series of statistical tests, including the coefficient of determination (R^2) value, a residual by predicted plot, an actual by predicted plot, and the distribution of the model fit error (MFE).

The first test, the R^2 value of the fit, indicates how large the sum of squared errors is in comparison to the total sum of squares. An R^2 value close to unity indicates that a large degree of the variability in the response (payload capability) is explained by the assumed second-order model. The R^2 is particularly relevant for this application because the number of degrees of freedom in the assumed model (six, from the number of β's in Eq. (3)) is much smaller than the number of points used to produce the fit. Furthermore, the points used in each fit were spread as uniformly as possible throughout the range of altitudes and inclinations examined. As will be discussed shortly, all R^2 values for the fits in this paper are greater than 0.971. The R^2 value for the Pegasus XL example is 0.9994.

The second test for the goodness of fit of the RSE is an actual by predicted plot, an example of which is shown in Fig. 3. The *x*-axis of Fig. 3 is the response value (i.e., payload capability) predicted by the RSE, and the *y*-axis is the actual response value (i.e., from the payload planner's guide). If the assumed model perfectly predicts the response, the actual by predicted plot is populated by a series of points along a one-to-one (45°) line of positive slope. Figure 3, the actual by predicted plot for the Pegasus XL, is an example of a good fit, as all points lie very close to the red one-to-one line and display little clumping.

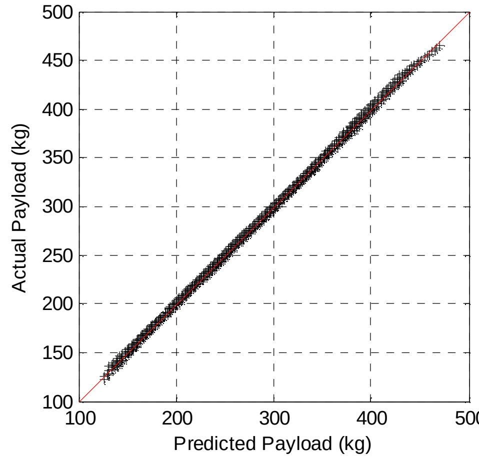

Figure 3. Actual by Predicted Plot for the Pegasus XL. *Note that data points lie very close to the 1:1 actual-to-predicted line with little clumping, indicating a suitable fit.*

The third test, similar in nature to the second test, is a residual by predicted plot, an example of which is shown in Fig. 4. The *x*-axis of the residual by predicted plot is identical to that of the actual by predicted plot. The *y*-axis, however, shows the difference between the actual and predicted response values. In the ideal case, if the payload capability of a launch vehicle is very precisely described by a second-order model, errors should be small and appear random without dependence on the predicted output. Note that in Fig. 4, which shows the Pegasus XL's residual by predicted plot, although some patterns are visible, the residual values are at most about 3% of the predicted values.

The fourth and final test is a check of the model fit error (MFE) distribution. This distribution represents the residuals as percentages of the actuals (i.e., as percent errors) and can be plotted as a histogram. Figure 5 shows this metric for the Pegasus XL; note that errors are distributed evenly across the 0% error line, indicating that the central tendency of the model has no upward or downward bias. Figure 6 shows the distribution of the absolute value of the error, and the 75th, 90th, 97.5th, 99.5th, and 100th (maximum) percentile errors are marked. Figure 6 shows, for example, that the 90th percentile error is 1.12% for the Pegasus XL. That is, the resultant RSE for the Pegasus XL correctly predicts the payload capacity to an accuracy within 1.12% for 90% of the points used to fit the model. The maximum error in the payload capacity prediction of the RSE is 3.12%. As a result, this may be regarded as a highly accurate model for applications that are able to tolerate 1-3% error in launch vehicle capacity (a 5.7 kg error at most). These percentiles will be reported shortly for the RSE fits of all launch vehicles considered.

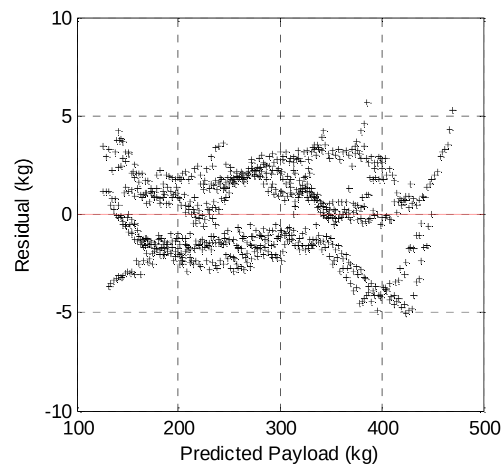

Figure 4. Residual vs. Predicted Values Plot. *Displays Pegasus XL's payload residual, or error between actual and predicted, versus the actual payload value from the payload planner's guide.*

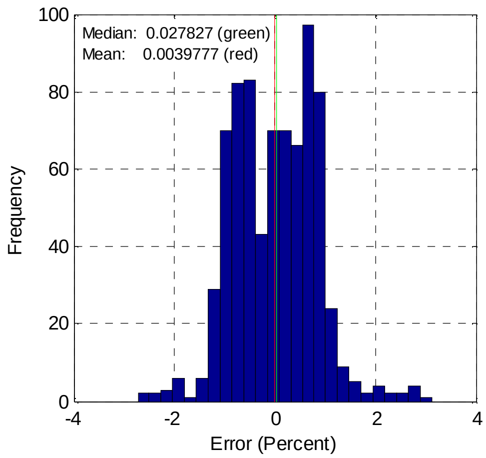

Figure 5. MFE Distribution for Pegasus XL. *Note that neither the mean nor median deviate from zero by more than 0.03%.*

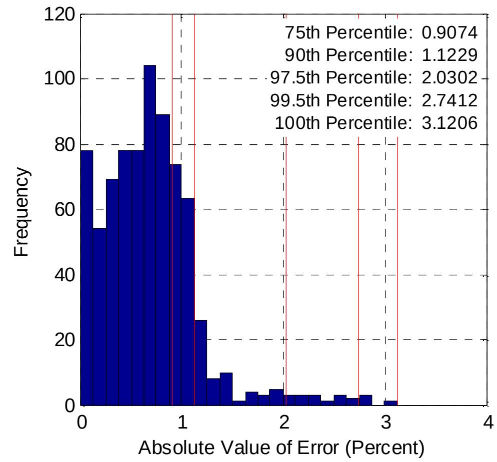

Figure 6. MFE Absolute Value Distribution for Pegasus XL. *Red lines indicate the 75th, 90th, 97.5th, 99.5th, and 100th percentile errors.*

III. Resulting Response Surface Equations

The fitting procedure described above is applied to different launch vehicles and launch vehicle families, including the Pegasus, Taurus, Minotaur, and Falcon series as well as the Delta IV, Atlas V, and the foreign Ariane and Soyuz vehicles. Derived and analyzed are 43 response surface equations. For example, the RSE for the Pegasus XL example, for which error statistics were provided in Section II, is given in Eq. (4):

$$m_{pay} = 531.7 - 0.2543 \cdot h - 0.8343 \cdot i - 2.136 \times 10^{-5} \cdot h^2 + 5.292 \times 10^{-4} \cdot i^2 - 0.008710 \cdot i \cdot h \quad (4)$$

Shown in Tables 1-3 are the error statistics and ranges of validity for all 43 response surfaces. Table 1 provides information for the launch vehicles manufactured by Orbital Sciences and SpaceX, Table 2 provides information for United Launch Alliance's (formerly Boeing's and Lockheed's) Delta IV and Atlas V launch vehicles, and Table 3 covers the foreign Ariane and Soyuz launch vehicles. Each table lists the coefficient of determination (R^2 value) of the fit, the number of data points used in the regression analysis, the model fit error (MFE) 90th and 97.5th percentiles, and the altitude and inclination ranges over which the fit is valid.[§]

As Tables 1-3 show, all R^2 values are at least 0.9715. The mean R^2 value for these 43 RSEs is significantly higher, at 0.9961. No fewer than 34 points were used for any one fit (more than five times as many as the minimum required to estimate the 6 parameters of the RSE). The mean number of data points used for fits is significantly higher, at 305. Tables 1-3 also show that the maximum 90th percentile model fit error (MFE) is 4.39% and the mean MFE 90th percentile is 0.97%. This is well within accuracy requirements for conceptual design and trade studies. Furthermore, if the Delta IV series is not considered, the maximum 90th percentile MFE is reduced to 2.06% and the mean 90th percentile MFE is reduced to 0.60%. This level of accuracy (a 90% probability of obtaining payload capability values correct to within 2%) is likely to also be applicable well after the stage of conceptual design.

The altitude and inclination ranges of validity are strongly dependent on the data available in the launch vehicle payload planner's guides. In general, altitude validity ranges extend from roughly 200 km low Earth orbits to 2000 km medium Earth orbits. Inclinations are more variable but generally span 28.5° to 90.0°. Except for the Falcon 1 and Falcon 1e vehicles, which provide data to inclinations up to 99.6°, circular retrograde orbits require extrapolation from these plots. More precise data and RSE fits for sun-synchronous circular orbits can be produced upon request.

[§] Specifically, these ranges indicate the span of inputs used in the regression. While small extrapolations may still be accurate, data does not exist in the payload planner's guides to validate the models outside of these ranges.

In general, these RSEs represent the aggregate of all circular orbit performance curves across multiple launch sites. The most common sites for the domestic vehicles are Cape Canaveral Air Force Station and Vandenberg Air Force Base, with limited additional capability from Kodiak Island, Wallops Flight Facility, and Reagan Test Site. Although payload planner's guides often associate an inclination capability with a single launch site, in some cases multiple launch sites are associated with the same inclination. In these cases, the higher-performing site is used for the regression. In certain rare cases, atypical launch trajectory features, such as inclination-specific dogleg maneuvers, would skew the entire fit for a launch vehicle. In these cases, which occur only for the Taurus and Minotaur vehicles, the discrepant inclination is excluded from the regression to avoid skewing predictions for other inclinations. Table 4 shows the error that results from applying the RSE (as documented in Table 1) to the excluded inclinations. For the Taurus, this applies to launches to 45°, and for the Minotaur this applies to the 64-65° inclination range. Note that the 90th percentile error on these excluded inclinations remains below 5%, and the 97.5th percentile errors remain below 7%.

Table 1. RSE Fit Statistics and Ranges of Validity for the Falcon, Minotaur, Pegasus, and Taurus Vehicles.

Vehicle	R^2	Data Points used in fit	RSE Model Fit Error		Altitude Range (km)		Inclination Range (deg.)	
			90th Percentile	97.5th Percentile	Min	Max	Min	Max
Falcon 9	0.9998	133	0.292%	0.373%	200	2000	28.5	90.0
Falcon 1e	0.9989	357	0.686%	1.150%	185	700	9.1	99.5
Falcon 1	0.9989	276	1.145%	1.948%	185	700	9.1	99.6
Minotaur I	0.9995	1194	1.317%	2.269%	186	1990	28.5	90.0
Minotaur IV	0.9950	838	2.059%	3.175%	185	1854	28.5	90.0
Pegasus XL without HAPS	0.9994	763	1.125%	2.071%	203	1393	11.0	90.0
Pegasus XL with HAPS	0.9990	770	1.219%	1.901%	501	1994	10.0	90.0
Taurus 2110	0.9937	909	1.796%	2.245%	200	1000	28.5	90.0
Taurus 2210	0.9922	995	1.794%	1.964%	200	1000	28.5	90.0
Taurus 3110	0.9980	874	1.175%	1.762%	200	1000	28.5	90.0
Taurus 3210	0.9950	899	1.808%	2.091%	200	1000	28.5	90.0

Table 2. RSE Fit Statistics and Ranges of Validity for the Delta IV and Atlas V Launch Vehicles.
Note that any R^2 values reported as 1.0000 are due to rounding.

Vehicle	R^2	Data Points used in fit	RSE Model Fit Error		Altitude Range (km)		Inclination Range (deg.)	
			90th Percentile	97.5th Percentile	Min	Max	Min	Max
Delta IV Medium	0.9833	399	4.195%	4.445%	388	4965	28.7	90.0
Delta IV Medium 4,2	0.9854	405	3.306%	3.557%	358	4987	28.7	90.0
Delta IV Medium 5,2	0.9848	385	3.602%	4.030%	348	4988	28.7	90.0
Delta IV Medium 5,4	0.9851	407	3.488%	4.554%	512	4982	28.7	90.0
Delta IV Heavy	0.9735	406	4.386%	4.822%	348	4982	28.7	90.0
Atlas V 401 Single-Burn	1.0000	35	0.051%	0.063%	204	498	63.4	90.0
Atlas V 401 Two-Burn	0.9997	144	0.124%	0.242%	519	1993	63.4	90.0
Atlas V 402 Single-Burn	0.9998	63	0.180%	0.246%	208	593	28.6	55.0
Atlas V 402 Two-Burn	0.9998	156	0.160%	0.220%	604	1983	28.6	55.0
Atlas V 411 Single-Burn	1.0000	44	0.043%	0.065%	202	497	63.4	90.0
Atlas V 411 Two-Burn	0.9997	168	0.122%	0.249%	499	1994	63.4	90.0
Atlas V 421 Single-Burn	1.0000	43	0.037%	0.060%	206	500	63.4	90.0
Atlas V 421 Two-Burn	0.9992	161	0.224%	0.379%	500	1996	63.4	90.0
Atlas V 431 Single-Burn	0.9972	156	0.465%	0.581%	208	497	63.4	90.0
Atlas V 431 Two-Burn	0.9967	169	0.489%	0.603%	501	1983	63.4	90.0
Atlas V 501 Single-Burn	0.9999	34	0.058%	0.063%	206	490	63.4	90.0
Atlas V 501 Two-Burn	0.9996	167	0.204%	0.358%	501	1998	63.4	90.0
Atlas V 511 Single-Burn	0.9999	36	0.058%	0.092%	208	499	63.4	90.0
Atlas V 511 Two-Burn	0.9999	176	0.092%	0.143%	503	1998	63.4	90.0
Atlas V 521 Single-Burn	0.9999	37	0.062%	0.107%	204	498	63.4	90.0
Atlas V 521 Two-Burn	1.0000	172	0.037%	0.054%	508	1992	63.4	90.0
Atlas V 531 Single-Burn	0.9993	54	0.159%	0.332%	187	554	28.6	90.0
Atlas V 531 Two-Burn	0.9998	167	0.165%	0.197%	501	1985	63.4	90.0
Atlas V 532 Single-Burn	0.9955	170	0.652%	0.812%	183	556	28.6	51.6
Atlas V 541 Single-Burn	0.9999	34	0.079%	0.100%	204	486	63.4	90.0
Atlas V 541 Two-Burn	0.9995	169	0.198%	0.281%	498	1991	63.4	90.0
Atlas V 551 Single-Burn	0.9999	35	0.069%	0.117%	204	487	63.4	90.0
Atlas V 551 Two-Burn	0.9998	168	0.128%	0.159%	502	1987	63.4	90.0
Atlas V 552 Single-Burn	0.9715	50	1.952%	2.736%	186	569	28.6	90.0
Atlas V 552 Two-Burn	0.9999	170	0.104%	0.153%	501	1356	63.4	90.0

Table 3. RSE Fit Statistics and Ranges of Validity for the Soyuz and Ariane Launch Vehicles.

Vehicle	R^2	Data Points used in fit	RSE Model Fit Error		Altitude Range (km)		Inclination Range (deg.)	
			90th Percentile	97.5th Percentile	Min	Max	Min	Max
Ariane 5-ES	0.9979	146	0.703%	1.102%	198	1392	48.0	86.0
Soyuz 2	0.9956	190	1.592%	1.741%	399	1567	51.8	90.0

Table 4. Model Error Statistics for Excluded Inclinations.

Vehicle	Excluded Inclination	Model Error	
		90^{th} Percentile	97.5^{th} Percentile
Minotaur I	64°	3.037%	5.114%
Minotaur IV	65°	2.005%	3.117%
Taurus 2110	45°	2.818%	5.687%
Taurus 2210	45°	4.437%	6.198%
Taurus 3110	45°	4.147%	4.463%
Taurus 3211	45°	4.591%	6.996%

IV. Example Applications

Presented next are three examples illustrating the practical use of the launch vehicle RSEs described above. Covered first is an example where an engineer has a defined payload and mission scenario and is faced with the task of selecting a launch vehicle that will provide sufficient margin for potential future mass growth. Second is an example where a payload mass is known and the RSEs are used to visualize the space of available orbits that different launch vehicles can provide. Third is an example of how an RSE can be mathematically differentiated to yield sensitivity information.

A. Deciding on a Launch Vehicle given Payload and Mission

One common task required in conceptual design is the selection of a launch vehicle once payload and orbit requirements are given. To illustrate how these RSEs may be used in such an application, we take as an example the 2006 TacSat-2 spacecraft, a joint effort among Department of Defense organizations and NASA. TacSat-2 was launched on December 16, 2006 from Wallops Flight Facility with the mission of both demonstrating responsive space capabilities and delivering 11 onboard instrument packages and experiments.[13,14] The mass of TacSat-2 was 370 kg, and it was launched aboard a Minotaur I rocket to a 40° inclined circular orbit with an altitude of approximately 410 km.[13,14]

Traditionally, a mission planner or engineer might refer to a payload planner's guide to analyze a candidate launch vehicle's capability at the desired orbit and inclination, as notionally illustrated in Fig. 7. This process would need to be repeated for each candidate launch vehicle and could require significant time and resources. In contrast, RSEs allow mission planners to use a program, such as Microsoft Excel or MATLAB, to calculate the payload capabilities automatically. In the example of TacSat-2, a user may calculate the payload capability to the 40°, 410 km circular orbit for several launch vehicles and compare launch margin (as well as cost and reliability, if this data is available). Any launch vehicles with a negative margin can be immediately eliminated from consideration.

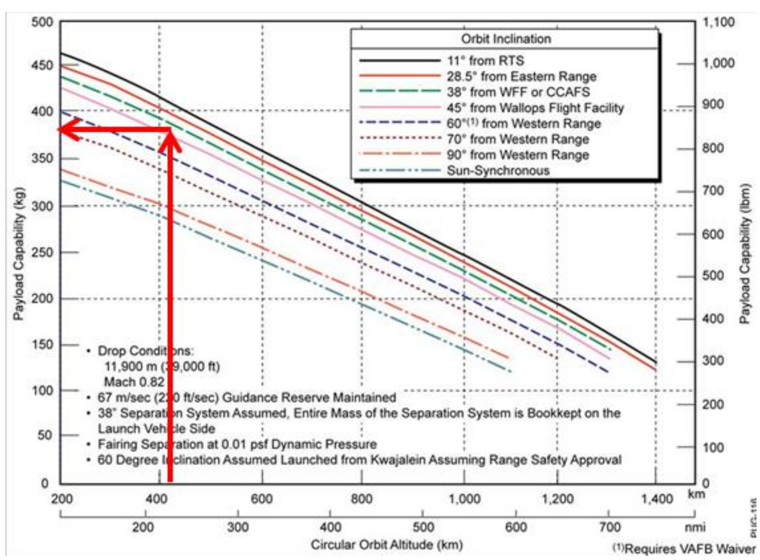

Figure 7. Notional TacSat-2 Graphical Payload Capability Lookup Technique for Pegasus XL (without HAPS; adapted from Ref. 1).

Table 5 shows the results of such RSE calculations, considering the Falcon 1, Falcon 1e, Minotaur I, Pegasus XL (without the Hydrazine Auxiliary Propulsion System, or HAPS), Taurus 2110, and Taurus 2210 as candidate launch vehicles. The second column in the table shows the payload capacity to the desired orbit using the RSEs described earlier. The third column shows the difference between the required and available payload capacity, and the fourth column shows this expressed as a percentage of the required capacity. Note that while the Falcon 1 would not be able to carry TacSat-2, the enhanced Falcon 1e would be able to do so with a 135% margin (e.g., enough margin to piggyback with another satellite, if desired). The Pegasus XL is able to carry TacSat-2 with a slight 4% margin. Note that the Pegasus XL with HAPS capability is not presented because its range of validity is limited to altitudes over 500 km. The Taurus 2110 and 2210 provide ample capability, although note that the Taurus 2210 has the slightly smaller capability because of its larger fairing. Finally the Minotaur I, on which TacSat-2 actually launched, demonstrates a modest 37% margin.

Table 5. RSE-Calculated Payload Capability and Margin for the TacSat-2.

Vehicle	Payload Capability (kg)	Margin (kg)	Margin (%)
Falcon 1	330.9	-39.1	-10.6%
Falcon 1e	869.1	499.1	134.9%
Minotaur I	507.2	137.2	37.1%
Pegasus XL without HAPS	385.2	15.2	4.1%
Taurus 2110	1176.7	806.7	218.0%
Taurus 2210	973.6	603.6	163.1%

B. Visualizing the Orbit Selection Space given Payload Constraints

While the previous example illustrated how these RSEs can be used to directly calculate payload capability from a known mission profile, another use involves visualization of the trade space of possible orbits during early stages of design. Using the RSEs, contour plots of payload as a function of altitude and inclination can be produced as in Fig. 8. The left plot of Fig. 8 shows the payload capability of the Minotaur I, and the right plot shows that of the Pegasus XL (without HAPS). Note that, as might be expected, both plots show a monotonic decrease in capability as altitude and inclination increase.

In this scenario, the required payload capability is known (370 kg, as with TacSat-2), and the regions of the plots that do not meet this constraint are shaded. The resulting white region not shaded is the feasible orbit trade space. Note that in the Minotaur plot, the user may maximize altitude to about 1000 km if choosing a low-inclination orbit or achieve a polar orbit if he accepts a lower 600 km altitude. Knowing the specification of this capability envelope is useful, for example, in trading the amount of global coverage against swath width or instrument field of view. For the Pegasus, the trade space is smaller; for example, 300 km in altitude may be gained if inclination is allowed to decrease from 80° to 28.5°. At the 200-500 km altitude range, this may represent a particularly important trade between global coverage (for a near-polar orbit) and spacecraft on-orbit lifetime (due to atmospheric drag).

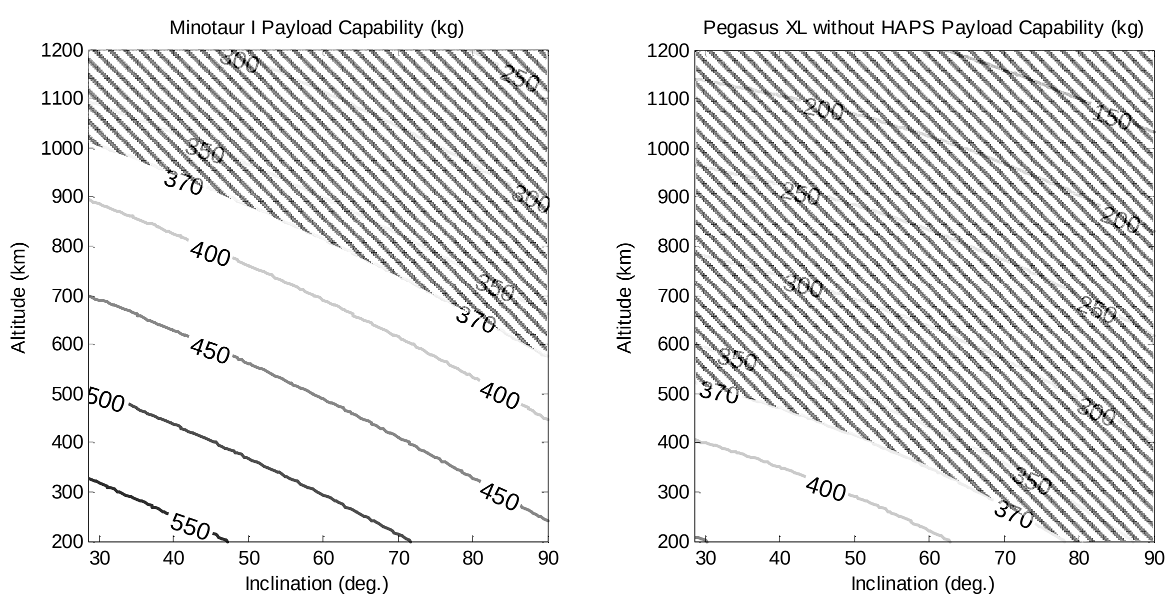

Figure 8. Minotaur I and Pegasus XL Payload Capability Contour Plots with 370 kg TacSat-2 Constraint.

C. Analytically Assessing Sensitivity to Mission Changes or Mass Growth

A final illustration of capabilities enabled by these RSEs is the analytical determination of payload sensitivity. Mass growth is a common concern during spacecraft design and development, and the analytical nature of the RSE provides a straightforward method of determining the effect of mass growth on mission altitude or inclination. The general equation for the fits in this study in Eq. (3) can be differentiated with respect to altitude (h) and inclination (i) to yield two partial derivatives of mass shown in Eqs. (5) and (6). Since the original RSE is quadratic, these partial derivatives are linear functions of altitude and inclination.

$$\frac{\partial m_{pay}}{\partial h} = \beta_1 + 2\beta_{11}h + \beta_{12}i \tag{5}$$

$$\frac{\partial m_{pay}}{\partial i} = \beta_2 + 2\beta_{22}i + \beta_{12}h \tag{6}$$

Equations (5) and (6) are plotted in Fig. 9 as a function of altitude and inclination for the Pegasus XL (without HAPS). While the sensitivity is a function of altitude and inclination, note that it is a relatively weak function in the case of the altitude sensitivity. Over the 1000 km altitude span and 60° inclination span of Fig. 9, the altitude sensitivity ranges from -0.29 to -0.22 kg/km (or equivalently, -4.6 to -3.4 km/kg). Thus, for every kilogram of mass added to a spacecraft launching on the Pegasus XL, the maximum attainable circular orbit altitude is reduced by 3.4 to 4.6 km if inclination remains constant. The inclination sensitivity is somewhat more variable, from -2.3 to -0.7 kg/deg (equivalently, -1.4 to -0.4 deg/kg). Thus, for every kilogram of mass added to a spacecraft launching on the Pegasus XL, the maximum attainable inclination is reduced by 0.4 to 1.4 degrees if altitude remains constant. Along with providing helpful rules of thumb, rapidly attainable sensitivity information such as this is well-suited for use in assessments of programmatic risk and launch vehicle robustness.

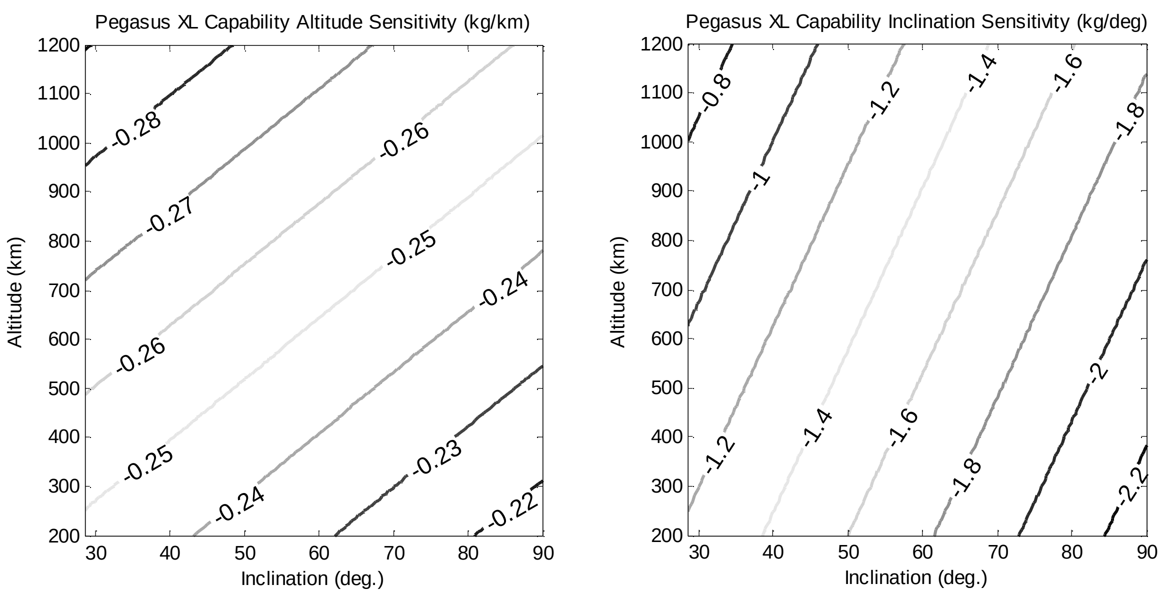

Figure 9. Pegasus XL Altitude Sensitivity ($\partial m_{pay}/\partial h$, left) and Inclination Sensitivity ($\partial m_{pay}/\partial i$, right).

IV. Conclusion

This paper presented the motivation, method, and statistical results for fitting a quadratic response surface to maximum payload capability for multiple launch vehicles, including the Pegasus, Taurus, Minotaur, and Falcon series as well as the Delta IV, Atlas V, and the foreign Ariane and Soyuz launch vehicles. The resulting RSEs were demonstrated to a model fit error no greater than 4.39% in the 90^{th} percentile and 4.82% in the 97.5^{th} percentile, with the mean model fit error across launch vehicles being much lower at 0.97% in the 90^{th} percentile and 1.25% in the 97.5^{th} percentile. Of the 43 RSEs generated, the minimum R^2 coefficient of determination is 0.9715 and the mean is 0.9961. As a result, these equations are sufficiently accurate and well-suited for use in conceptual design and beyond, enabling rapid trade studies among a variety of orbit altitudes, inclinations, and launch vehicle options. Examples of such trades were provided, including demonstrations using the RSEs to (1) select a launch vehicle given an orbit inclination and altitude, (2) visualize orbit constraints and trades given a spacecraft mass, and (3) analytically calculate sensitivity of orbital parameters to mass growth.

Areas for future expansion of our current RSE work include (1) addition of new launch vehicles such as the Proton and other foreign launch vehicles, (2) addition of response surfaces for interplanetary, geosynchronous transfer orbit, and sun-synchronous orbit scenarios, and (3) creation of a user interface to facilitate efficient and transparent use of the database.

Whether in its current state or with future expansion, our RSE work provides a powerful tool to the engineer during conceptual design and beyond. It is intended that the capabilities enabled by this work will aid the engineer or project manager in making efficient, informed trades and decisions on launch options early during design and development phases for a wide variety of mission applications.

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