# Decentralized Mean Orbit-Element Formation Guidance, Navigation, and Control: Part 2

Jay W. McMahon* and Marcus J. Holzinger †

**This paper is the second of a two-part discussion on a Formation Flight Guidance, Navigation, and Control solution that introduces a decentralized formation control approach. The previously defined formation framework is expanded and considerations for formation design are discussed, including $J_2$ invariance and collision avoidance. Several mean orbit element controllers are discussed that control the relative formation in mean orbit element space. These controllers are tested with perfect navigation, and combined with the previously developed consensus algorithm. Discussion on the relative performance of the combined control and consensus simulation compared to the control with perfect navigation is presented. This is a key issue in the future use of a control scheme on a distributed formation.**

## I. Introduction

This paper is the second portion of a two-part discussion on a Formation Flight Guidance, Navigation, and Control solution that introduces a decentralized formation control approach. The first paper presented a formation framework, which outlines a formation definition, and the methodology and performance for the individual spacecraft of a formation coming to a consensus on the location of a weighted orbital element barycenter. The goal of this paper is to demonstrate formation stability and controllability while incorporating the consensus process on the formation barycenter.

In the first paper, a method for reaching consensus on a weighted orbital element barycenter is given. Here, we discuss how the choice of weighting is made to fulfill different formation mission requirements. The simple case of one spacecraft acting as the chief will have that spacecrafts weighting set to 1 with all others set to zero, therefore consensus will be found on the chief spacecrafts mean orbital elements. In particular, we explore how changing the weightings can affect the performance of the formation as a whole; we are especially interested in how the relative positions change with a different barycenter, and how fuel performance of keeping the formation stable can be affected by the choice of barycenter.

As outlined in Part I, each spacecrafts slot in the formation is defined by a set of differential mean elements with respect to the chosen barycenter. Controlling the mean orbital elements allows us to ignore the short-term variations in the osculating elements, thus conserving fuel. The differential mean orbital elements drift due to secular perturbations, and therefore must be controlled in some manner. Intelligent design of the formation can minimize some secular perturbations by defining certain differential orbital element relationships, but in all cases control is required to ensure that the desired relationships are maintained. This paper demonstrates the feasibility of using differential mean orbit element control to stabilize the desired formations under the influence of $J_2$.

Demonstration of control of the formation within this framework is necessary, however the stability of this control requires perfect knowledge of the barycenters orbital elements. Realistic implementation of these methods must combine the issue of formation control with the problem of properly determining the

*Research Associate, Aerospace Engineering Sciences, University of Colorado - Boulder, AIAA Member
†Assistant Professor, School of Aerospace Engineering, Georgia Institute of Technology, AIAA Member

barycenter of the formation through consensus. The implications for the formation stability are explored, and comparisons of the performance to the nominal case are given.

We note here that there is a wide and active discussion in the literature about different formation control methodologies. In this work, we make no attempt to choose the "best" or an optimal controller. We implement the continuous mean orbit element controller developed by Schaub and colleagues.[3] Even these authors have multiple methods of controlling the mean orbit elements,[5,6] and again there is no technical metric used in the control choice here other than that it has been designed to control mean orbit elements.

To summarize, there are three primary contributions in this paper: 1) relative formation design and desired consensus weightings are discussed based on formation goals such as minimizing fuel usage; 2) two differential orbit element controls are applied to the formation framework defined in Part 1; and 3) the formation control performance is analyzed under the influence of both consensus on the weighted orbit element barycenter and differential orbit element control. Conclusions and future work are discussed.

## II. Consensus Weighted Formation

Initially, in is important to understand the implications of the consensus weighted formation defined in Part 1[1] for the formation dynamics and control. First we note from the definition of the weighted formation barycenter (reprinted here for convenience),

$$\bar{\boldsymbol{\text{œ}}}_k^b = \sum_{i=1}^{N_f} w_k^i \bar{\boldsymbol{\text{œ}}}_k^{b,i} = \sum_{i=1}^{N_f} w_k^i \left(\bar{\boldsymbol{\text{œ}}}_k^i - \delta\bar{\boldsymbol{\text{œ}}}_{r,k}^i\right) \tag{1}$$

that the weighted formation barycenter must be on the convex hull of the individual craft barycenters, $\bar{\boldsymbol{\text{œ}}}_k^{b,i}$. The individual craft barycenters are determined from the currently estimated mean orbit elements of the craft, $\bar{\boldsymbol{\text{œ}}}_k^i$, and the craft's formation slot, $\delta\bar{\boldsymbol{\text{œ}}}_{r,k}^i$, as shown from the second equality.

The weightings are subject to the following constraint

$$\sum_{i=1}^{N_f} w_k^i = 1 \tag{2}$$

which implies a corresponding constraint on the rate of change of the weightings such that

$$\sum_{i=1}^{N_f} \dot{w}_k^i = 0 \tag{3}$$

The consensus perspective is that if all craft have good information content, the weightings will converge to equal values

$$w^i \to \frac{1}{N_f} \tag{4}$$

which means that the weighted barycenter will be at the average of where each craft believes the barycenter is located. Alternatively, it is possible that only one spacecraft's information can be trusted and therefore that spacecraft's weighting would be set to 1. In this case, all of the other spacecraft would use the single craft's information as their weighted barycenters as well, which will influence the control solutions computed as discussed in the following section. An illustration of the weighted barycenter is shown in Fig. 1.

If the consensus problem is successful, the individual craft barycenters will converge to the same value. When this occurs, the weightings can be changed to any valid combination and the weighted barycenter will stay the same. This allows for some flexibility in the formation definition. Implications of changing the weightings for the controls are discussed in Section III.

There are two important benefits that can be derived from changing a spacecraft's weighting. First, a spacecraft's fuel usage for maintaining a formation is minimized if it has a weighting of 1. This is true because

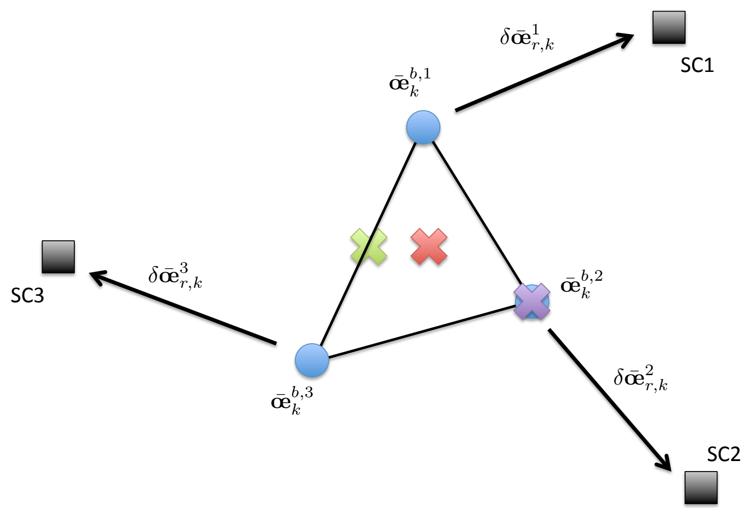



Figure 1: Illustration of the weighted barycenter on the convex hull of the individual craft barycenters (shown as blue circles). Three cases are shown: for weightings $w^1 = w^2 = w^3 = 1/3$, the weighted barycenter would be at the red x; for weightings $w^1 = w^3 = 1/2$ and $w^2 = 0$, the weighted barycenter would be at the green x; for weightings $w^1 = w^3 = 0$ and $w^2 = 1$, the weighted barycenter would be at the purple x.

the spacecraft will not use any fuel to move its own barycenter to agree with the weighted barycenter; this agreement is always true. If the formation barycenter is free to drift (see Section A), then having a weighting of 1 will stop the spacecraft from burning at all for formation keeping purposes.

The second implication of changing the spacecraft consensus weightings occurs for formation reconfiguration. In the case where spacecraft $n$ should be moved to a new slot, its weighting is set $w^n = 0$, and the formation slot is modified as necessary. While the craft moves to the new slot, its weighting is left $w^n_k = 0$ so that it has no influence on the location of the formation barycenter. During this time, this spacecraft will move to its formation slot with respect to the weighted barycenter that does not include its own position. Once spacecraft $n$ has reached its new formation slot, then all of the weightings can again be rearranged as desired because this spacecraft will now have zero formation error, and so giving it a non-zero weighting will not move the barycenter as discussed above.

In light of these example dependences on the formation slot, it is important to understand how design of the formation slots in general can be used for different outcomes. First and foremost, as has been alluded to in several places, the formation should be designed so there is no secular drift between spacecraft due to $J_2$, which results in the following Lemma.

**Lemma II.1.** $J_2$ ***Invariant Formation Slots***
*Given a weighted formation barycenter, $\bar{\text{œ}}^b_k$, a $J_2$ invariant formation slot has 4 degrees-of-freedom.*
***Proof:*** *A $J_2$ invariant relative orbit is required to meet two constraints*[2]

$$\delta a = 2Da\delta\eta \tag{5}$$

$$\delta\eta = \frac{\eta}{4}\tan i\delta i \tag{6}$$

*where $\eta = \sqrt{1-e^2}$ and*

$$D = \frac{J_2}{4L^4\eta^5}(4+3\eta)(1+5\cos^2 i) \tag{7}$$

*where $L = \sqrt{a/R_{eq}}$, $R_{eq}$ is the Earth's equatorial radius, the orbit elements are components of $\bar{\text{œ}}^b_k$, and the differential orbit elements are components of $\delta\bar{\text{œ}}^i_{r,k}$. Since the formation slot is a six-dimensional vector,*

*imposing two constraints on its members leaves 4 degrees-of-freedom.*
□

The values of $\delta\omega$, $\delta\Omega$, $\delta M$ are arbitrary as they do not appear in the above constraints. Also, one of $\delta a$, $\delta e$, or $\delta i$ can be set arbitrarily, with the other two values being obtained from constraints. Note that the constraints shown in Eqs. (5) and (6) assume a small $\delta L$; for higher eccentricities and near equatorial orbits, the non-simplified relationships derived by Schaub[?] should be used to compute a $J_2$ invariant formation slot.

There is a subtlety to choosing a $J_2$ invariant formation that is not immediately obvious. In general, you can not arbitrarily pick a weighted formation barycenter and $J_2$ invariant orbits for the entire formation. This occurs because we apply multiple dependent constraints to the formation slots (in $a$, $e$ and $i$) and the barycenter with are not consistent. Specifically to meet the desired weighted barycenter with given weighting we have

$$a^b = \sum_i^{N_f} (a^b + \delta a^i) w^i \tag{8}$$

$$e^b = \sum_i^{N_f} (e^b + \delta e^i) w^i \tag{9}$$

$$i^b = \sum_i^{N_f} (i^b + \delta i^i) w^i \tag{10}$$

The weights sum to one, so these constraints reduce to

$$0 = \sum_i^{N_f} \delta a^i \cdot w^i \tag{11}$$

$$0 = \sum_i^{N_f} \delta e^i \cdot w^i \tag{12}$$

$$0 = \sum_i^{N_f} \delta i^i \cdot w^i \tag{13}$$

Alone, these constraints can easily be satisfied. However, the addition of the $J_2$ invariance adds two non-linear constraints for each spacecraft of the form

$$\delta a^i = f_a(\delta e^i, a^b, e^b, i^b) \tag{14}$$

$$\delta i^i = f_i(\delta e^i, a^b, e^b, i^b) \tag{15}$$

due to the fact that $\delta e$ is non-linearly related to $\delta\eta$. Combining all of these constraints gives a total of $3 + 2N_f$ constraints for $3 + 3N_f$ variables (the barycenter and formation slots), giving hope that there will be a solution. Upon combining the constraints, however, we find that the combination of the non-linear constraints from Lemma II.1 are not consistent with the linear barycenter constraints for all three elements, and therefore they can not all be satisfied. The relationship between $\delta a$ and $\delta i$ is linear, as is seen by combining Eqs. (5) and (6), and so it is consistent with the constraints in Eq. (11) and (13).

The exception to this is in the case where we have the classical chief spacecraft $k$, with $w^k = 1$ and $\delta a^k = \delta e^k = \delta i^k = 0$. In this case, the weighted barycenter is always located wherever this craft finds itself in mean element space.

Having pointed this subtlety out, we note that after iterating between the $J_2$ invariance offset from a desired orbit barycenter and the weighted barycenter of the resulting $J_2$ invariant slots, the differences can be made quite small for modest formation slots, so that the errors in the formation design due to this issue do not greatly impact control performance. We recommend choosing the formation slots and barycenter to minimize the $J_2$ induced drift rates in the node and mean latitude, which in turn minimizes the amount of control effort needed to keep the formation stable.

After considering the constraints for a $J_2$ invariant formation, the 4 available degrees-of-freedom leads to the following corollary.

**Corollary II.1.** ***Passively Safe Formation Slots***
*A $J_2$ invariant formation slot can be made passively safe to collisions with any neighboring formation slots with a minimum of 1 degree-of-freedom remaining.*
***Proof:*** *From Lemma II.1, it is known that a $J_2$ invariant formation slot has 4 degrees-of-freedom. The most restrictive set of constraints to avoid any neighboring formation slot (another $J_2$ invariant formation slot with respect to the same barycenter) will apply three further constraints to a formation slot. Therefore one degree-of-freedom remains.*
□

The most restrictive set of constraints will require the formation slot to avoid a 3-dimensional volume of space corresponding to the possible combined positions of the neighboring formation slots. It is entirely possible, however, that for a given set of formation slots, passive safety may only impose zero, one, or two additional independent constraints of the $J_2$ invariant formation slot. Therefore at a minimum there will be one degree-of-freedom available for design purposes, however it is also possible that the full four degrees-of-freedom will still be available after ensuring passive safety constraints. Typically this type of constraint will be defined in the Hill frame position coordinates, and will then be transformed into mean orbit element space.

## III. Differential Mean Orbit Element Controller

Mean orbit element controller so that we do not use energy to reject periodic variations in the osculating orbit elements caused by $J_2$. Also, since the formation is designed to be $J_2$ invariant by assigning the correct mean orbital elements, this is what we want to control.

The controller implemented for this study is the mean orbit element controller outlined in.[?] The control law that ensures convergence of the formation slot errors to zero follows the law

$$[B(\bar{\boldsymbol{\text{œ}}}^i_k)]\mathbf{u}^i_k = -[A(\bar{\boldsymbol{\text{œ}}}^i_k)] + [A(\bar{\boldsymbol{\text{œ}}}^i_{r,k})] - [P]\delta\bar{\mathbf{e}}^i_k \tag{16}$$

where the $[B]$ matrix represents Gauss' variational equations, $\mathbf{u}^i_k$ is the control acceleration applied to spacecraft $i$ at time $k$, the $[A]$ matrix represents the change in the mean orbital elements due to $J_2$ derived from Brouwer's first order theory,[?] and the $[P]$ matrix is a diagonal positive definite matrix with entries changing based on the true anomaly and latitude, as defined in.[?] The reference mean orbit elements for spacecraft $i$, $\bar{\boldsymbol{\text{œ}}}^i_{r,k}$, are defined according the desired control; several options are discussed in the following sections. The orbit element error is then defined as

$$\delta\bar{\mathbf{e}}^i_k = \bar{\boldsymbol{\text{œ}}}^i_k - \bar{\boldsymbol{\text{œ}}}^i_{r,k} \tag{17}$$

The desired control is determined by computing the pseudo-inverse of the $[B]$ matrix at each time $k$ and pre-multiplying both sides of Eq. (16) by this in order to determine the best control acceleration in a least-squares sense. Although this process does not guarantee that the stability criteria satisfied by the law in Eq. (16) is strictly satisfied, good performance for formation station-keeping has been found.

One draw-back of this controller is that it is not optimized in any strict sense to minimize fuel usage. The gains have been manually tuned to give good performance, and to preclude attempts to null errors at points in the orbit where it is hard/impossible to do. However there is no guarantee of optimality of these gains. In the future, more fuel efficient methods of station-keeping should be found. Also, since this control law is continuous, it will always fire to some degree since the errors are not perfectly nulled out. Therefore some fuel savings can be realized by putting a dead-band on the controller so it will not fire if errors are below a given bound.

### A. Formation Control

In this case, the only goal is to keep the spacecraft in relative formation. This means that the location of the weighted barycenter is important only for defining where each formation slot is in mean element space, however the barycenter itself is *not* controlled.

In this case, the desired mean orbit elements for each craft is given by

$$\bar{\text{œ}}^i_{r,k} = \bar{\text{œ}}^b_k + \delta\bar{\text{œ}}^i_{r,k} \tag{18}$$

which is the consensus weighted barycenter plus the formation slot. The error vector then becomes

$$\begin{aligned} \delta\bar{\mathbf{e}}^i_k &= \bar{\text{œ}}^i_k - \bar{\text{œ}}^b_k - \delta\bar{\text{œ}}^i_{r,k} &(19)\\ &= \bar{\text{œ}}^{b,i}_k - \bar{\text{œ}}^b_k &(20)\end{aligned}$$

This is the difference between where the current spacecraft thinks the barycenter is, and where the consensus solution is. In other words, the controller will try to move the current spacecraft's barycenter to the consensus solution.

The difficulty with this controller is that as each spacecraft attempts to move its barycenter to the consensus solution, the consensus solution moves! The weighted barycenter dynamics are given by the time derivative of Eq. (1)

$$\dot{\bar{\text{œ}}}^b_k = \sum_{i=1}^{N_f} \left[\dot{w}^i_k \left(\bar{\text{œ}}^i_k - \delta\bar{\text{œ}}^i_{r,k}\right) + w^i_k \left(\dot{\bar{\text{œ}}}^i_k - \delta\dot{\bar{\text{œ}}}^i_{r,k}\right)\right] \tag{21}$$

In this case we assume the weightings and the formation slots are held constant so that the weighted barycenter dynamics simplify to

$$\dot{\bar{\text{œ}}}^b_k = \sum_{i=1}^{N_f} w^i_k \dot{\bar{\text{œ}}}^i_k \tag{22}$$

The motion of the weighted barycenter is the weighted sum of the motion of each spacecraft since the formation slots are constant offsets. The dynamics of each spacecraft's mean state is given by

$$\dot{\bar{\text{œ}}}^i_k = [A(\bar{\text{œ}}^i_k)] + [B(\bar{\text{œ}}^i_k)]\mathbf{u}^i_k \tag{23}$$

This tells us that if there is no control in the system, the weighted barycenter dynamics are given by

$$\dot{\bar{\text{œ}}}^b_k = \sum_{i=1}^{N_f} w^i_k [A(\bar{\text{œ}}^i_k)] \tag{24}$$

which is the weighted drift of each spacecraft. This enforces the fact that under the natural dynamics the weighted barycenter will stay at the same location in the convex hull of the individual spacecraft barycenters.

It is interesting to note here that if the formation is in a $J_2$ invariant condition at this point, then the barycenter will stay in this configuration without control since its drift rates will equal those of spacecraft in formation. However, if the barycenter has moved away from the designed $J_2$ invariant point in mean element space during the controlled period, then the formation as a whole will no longer be $J_2$ invariant.

If the controller is executed perfectly as per Eq. (16), the weighted barycenter dynamics are given by

$$\dot{\bar{\text{œ}}}^b_k = \sum_{i=1}^{N_f} w^i_k \left([A(\bar{\text{œ}}^i_{r,k})] - [P]\delta\bar{\mathbf{e}}^i_k\right) \tag{25}$$

Now we see that as $\delta\bar{\mathbf{e}}^i_k \to \mathbf{0}$, the dynamics approach those in Eq. (24) because this implies that the spacecraft state is approaching the reference state. However we know that due to the usage of the pseudo-inverse in the control execution, these dynamics will not occur precisely.

At this point it is clear that unless everything is perfect to begin with, this controller will allow the barycenter to drift from the design point, which will in turn break the formation's designed $J_2$ invariance. As long as there is enough fuel and control authority, this controller will keep the spacecraft in the designed relative formation, but the station-keeping costs will be higher due to the fact that the $J_2$ invariance is lost, and therefore the controller has to fight these errors.

In light of this issue, we propose adding adaptive formation slot control. Simply put, once the formation is controlled so that the mean orbit element slot errors for each spacecraft are below some arbitrary limit, each spacecraft will be allowed to adjust is formation slot to a $J_2$ invariant slot around the current weighted barycenter. A secondary check on the current $J_2$ invariance error will also be made so that if the formation is already $J_2$ invariant this controller is not repeatedly executed. The adaptive formation slot control is executed as follows for each spacecraft. Each of the three possible $J_2$ invariant formation slots for the spacecraft are computed by fixing alternately $\delta a$, $\delta e$, or $\delta i$ to be the same as the current formation slot, but the resulting constraints on the other two are now computed with the current weighted mean barycenter mean orbit elements. The new formation slot is chosen to be the $J_2$ invariant slot that is closest to the current formation slot, in terms of fractional change from the current slot. Mathematically speaking, the distance from the current slot is determined by (in the case of keeping the inclination fixed)

$$d_s = \left| \frac{\delta a_n - \delta a_r}{\delta a_r} \right| + \left| \frac{\delta e_n - \delta e_r}{\delta e_r} \right| \tag{26}$$

where the subscript "n" refers to the new slot value, and the subscript "r" refers to the previous reference value. This distance is computed for the constant $\delta a$ and constant $\delta e$ cases, and the smallest of the three values is used as the new formation slot. Implementing this adaptive control will balance the goals of keeping fuel expenditures low while keep the formation as close to originally designed as possible. One caveat to this method, however, is that this could lead to a long term drift of the relative formation as the barycenter moves over time. Therefore it may be necessary, depending on the goals of the particular formation, to reset the formation to effectively reset this long term drift.

### B. Barycenter Control

The previous controller will ensure that the relative formation is kept, but it does nothing to control the location of the barycenter. It is possible, however, that a mission could require that the formation be kept in a relative sense, and located at a specific inertial location in time. In this section, we modify the control methodology to accomplish this goal.

In this case, the desired mean orbit elements for each craft is given by

$$\bar{\boldsymbol{\text{œ}}}^i_{r,k} = \bar{\boldsymbol{\text{œ}}}^b_{r,k} + \delta\bar{\boldsymbol{\text{œ}}}^i_{r,k} \tag{27}$$

which is the designed weighted barycenter plus the formation slot. Note that the designed mean orbit element barycenter changes in time even for a $J_2$ invariant orbit as there are secular rates in $\omega$, $\Omega$, and $M$. Recall that the $J_2$ invariant orbit simply tries to match these rates. Using this definition for the controller will cause both the relative formation, and the location of the formation in space, to be controlled simultaneously.

Eq. (21) shows that the change location of the weighted formation barycenter is controlled by changing the weightings, the formation slots, and the current mean orbit elements of the spacecraft in the formation. The proposed controller affects the barycenter location control by moving each craft individually. However, if we further consider the likely case where the spacecraft are in their respective formation slots, but the barycenter is not located in the desired position, the barycenter can be moved by changing the the weightings and formation slots. Assuming that the same relative formation is desired, this rules out changing the formation slots. Therefore we examine changing the barycenter though changing the weightings in combination with the spacecraft orbit elements.

Changes in the weightings are generally arbitrary, subject to the constraint in Eq. (3). As discussed previously, however, if the formation is well controlled, so that all spacecraft are in their formation slots, then the individual craft barycenters are equal to the same constant value, $\bar{\text{œ}}_k^{b,i} = \mathbf{C}\ \forall i$. Therefore if only the weightings are changed on a settled formation,

$$\dot{\bar{\text{œ}}}_k^b = \mathbf{C} \sum_{i=1}^{N_f} \dot{w}_k^i = 0 \tag{28}$$

In other words, the weightings can be arbitrarily changed without creating a rate on the weighted orbit barycenter.

One of the easiest ways to control the barycenter would be to change the weightings so one spacecraft has $w^i = 1$, and then control that spacecrafts orbital elements so that the barycenter moves to the desired location. Once the formation is settled around the new barycenter, the weightings can be changed again to arbitrary values if desired. The same controller from above can be used to directly modify a spacecraft's mean orbit elements by changing the reference barycenter to a new designed barycenter state to move the formation.

This method allows us to control the relative formation, as well as the orbit barycenter location, while only needing to trust the mean orbital elements of one craft in the formation at a minimum. This may be useful for situations where only one craft has inertial measurements and the rest have only relative measurements. Based on Eq. (21), it is also possible to move the formation barycenter by assigning mean orbit element sets to multiple spacecraft in the same manner; only using one spacecraft is simply an easy illustration.

### C. Formation Control with Consensus

The key to implementing this controller in parallel with the consensus process relies on the separation theorem implications, which tell us that if the estimator (in this case the consensus filter) can converge on a solution at a faster rate that the controls are implemented, then the combined system will be controlled to the consensus solution.[4] This does not imply that no errors will occur. In fact, if the consensus process finds a formation barycenter that is not exactly correct, this error will propagate into the controller.

The take away from this realization is that if we can determine that there is a high uncertainty in the consensus solution, the spacecraft will not execute maneuvers. The reasoning is simple - if we are unsure we are going to improve the state, or even exactly how we will change the state, there is no reason to use fuel to do so. This implies that the formation stability is not ensured. However, once a quality consensus solution is found, the controller is enabled and any formation slot errors are then nulled.

## IV. Simulation Test Cases

This section presents and discusses the results of four simulation test cases that were run to illustrate the performance of the controllers discussed in Section III. The first case uses the formation controller (from Section III.A) with perfect navigation. Likewise the second case uses the barycenter controller (from Section III.B) with perfect navigation. The next two cases are similar to the first two in the controllers, however the consensus algorithm from Part I[1] is now used to provide more realistic navigation information to the controllers.

A number of aspects of the simulation are kept the same between the different test cases. First and foremost, each formation consists of three spacecraft. The nominal design barycenter is

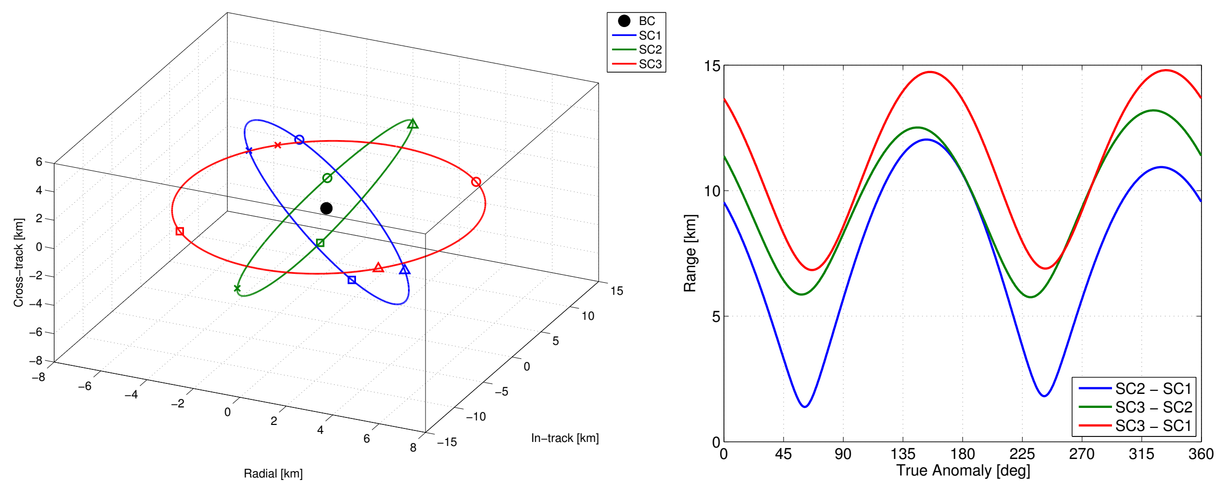
$$\bar{\text{œ}}^{b,d} = [R_{eq} + 500 \text{ km} \quad 0.05 \quad 28^\circ \quad 30^\circ \quad 45^\circ \quad 150^\circ]^T \tag{29}$$

The desired formation slots were made arbitrarily by setting $\delta i$, $\delta\omega$, $\delta\Omega$, and $\delta M$ for each of the spacecraft in the formation, and computing $\delta a$ and $\delta e$ to make the formation slots invariant to the designed barycenter in Eq. (29). The formation slots that are used for the simulations are given in Table 1.

The nominal formation is illustrated in Fig. 2(a) in the Hill frame with respect to the nominal designed barycenter. Fig. 2(b) shows the range between each of the spacecraft over the course of one orbit, which verifies that our arbitrary formation doesn't nominally lead to any collisions between the spacecraft.

Table 1: Desired Formation Slots for Nominal Simulations (* = designed for $J_2$ invariance)

| $\delta\bar{\text{œ}}^i_{r,0}$ | SC1 | SC2 | SC3 |
|---|---|---|---|
| $\delta a$ | -2.558* m | -2.558* m | 0.0255* m |
| $\delta e$ | 4.607e-4* | 4.607e-4* | 4.628e-6* |
| $\delta i$ | $0.01°$ | $0.01°$ | $0.0001°$ |
| $\delta\omega$ | $0.1\cos(\bar{\text{œ}}^b(3))°$ | $-0.33\cos(\bar{\text{œ}}^b(3))°$ | $-1°$ |
| $\delta\Omega$ | $-0.1°$ | $0.1°$ | $0°$ |
| $\delta M$ | $0°$ | $0.2°$ | $1°$ |




(a) Hill frame

(b) Ranges

Figure 2: The nominally designed formation given in Table 1 with respect to the barycenter in Eq. (29). The position of the craft in the Hill frame at true anomalies of 45°, 135°, 225°, 315° are shown by the circle, triangle, square and x symbols, respectively.

Each simulation also used equal weightings for each of the spacecraft,

$$w_1 = w_2 = w_3 = 1/3 \tag{30}$$

The equal weightings were used because it illustrates a formation without an actual chief spacecraft. Although we previously discussed reasons for varying the spacecraft weightings, these applications are not illustrated in this particular study as they are not necessary to illustrate the basic formation navigation and control issues central to this paper.

At this point, we briefly digress to discuss the issue of the designed barycenter versus the weighted barycenter and $J_2$ invariance, as discussed in Section II. Some quick algebra shows that the weighted barycenter of the formation slots in Table 1 differs from the designed barycenter by

$$\delta\bar{\text{œ}}^b = [-1.714 \text{ m} \quad -0.0017 \quad 0.0067^\circ \quad -0.401^\circ \quad 0^\circ \quad 0.400^\circ]^T \tag{31}$$

Compared to the designed barycenter, these differences are small. More importantly, we can check to see how far the formation slots are from the $J_2$ invariance conditions of the weighted barycenter. SC1 and SC2 had the same inclination offset to design the $J_2$ invariance, and compared to the actual weighted mean barycenter their formation slots are 6.1092e-4 m off in $\delta a$, and 2.6869e-6 off in $\delta e$. SC3 has errors of 6.1092e-6 m off in $\delta a$, and 2.7241e-8 off in $\delta e$. These errors are orders of magnitude less than the actual formation slots, implying that this difference is probably insignificant, especially once navigation and control errors are considered.

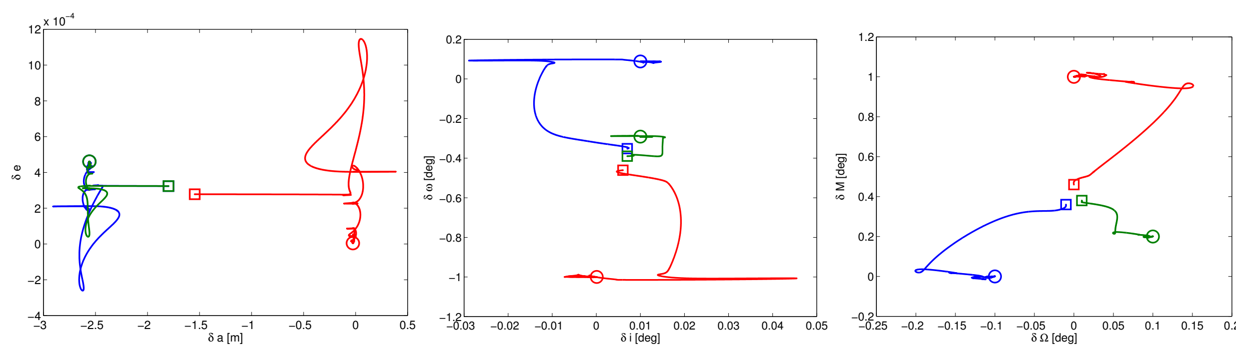
Finally, the nominal gains used in each case for the controller are those given by Schaub.[3]

Results from each of the test cases are discussed in turn in the following sections.

### A. Formation Control with Perfect Navigation

In this Section we run a test case with the formation controller with perfect navigation information. The initial condition for each spacecraft is 10% of the desired formation slot error given in Table 1 - e.g. $\delta a_1 = -0.2558$ m. These initial conditions can be thought of as the spacecraft moving from deployment out to desired formation slots. The simulation was run for 10 Earth orbits, which is plenty of time to show convergence to the desired formation.

The mean orbit element errors are shown in two views in Figs. 3 and 4. It is clear from these plots that the controller does a good job of moving each spacecraft to its desired formation slot. The errors in all orbit elements for each of the three spacecraft are effectively zeroed after 4-5 orbits.




(a) $\delta a$ - $\delta e$ (b) $\delta i$ - $\delta\omega$ (c) $\delta\Omega$ - $\delta M$

Figure 3: Spacecraft trajectories in differential mean orbit element space, shown as three 2-dimensional subspaces. The squares signify the starting position, and the circles are the targeted formation slots. Note that SC1 and SC2 have the same starting and targeted locations in the $\delta a - \delta e$ subspace.

The spacecraft trajectories in the Hill frame with respect to the weighted barycenter are shown in Fig. 5. Significant deviations from the desired formation slots are see as the spacecraft move outward from near the

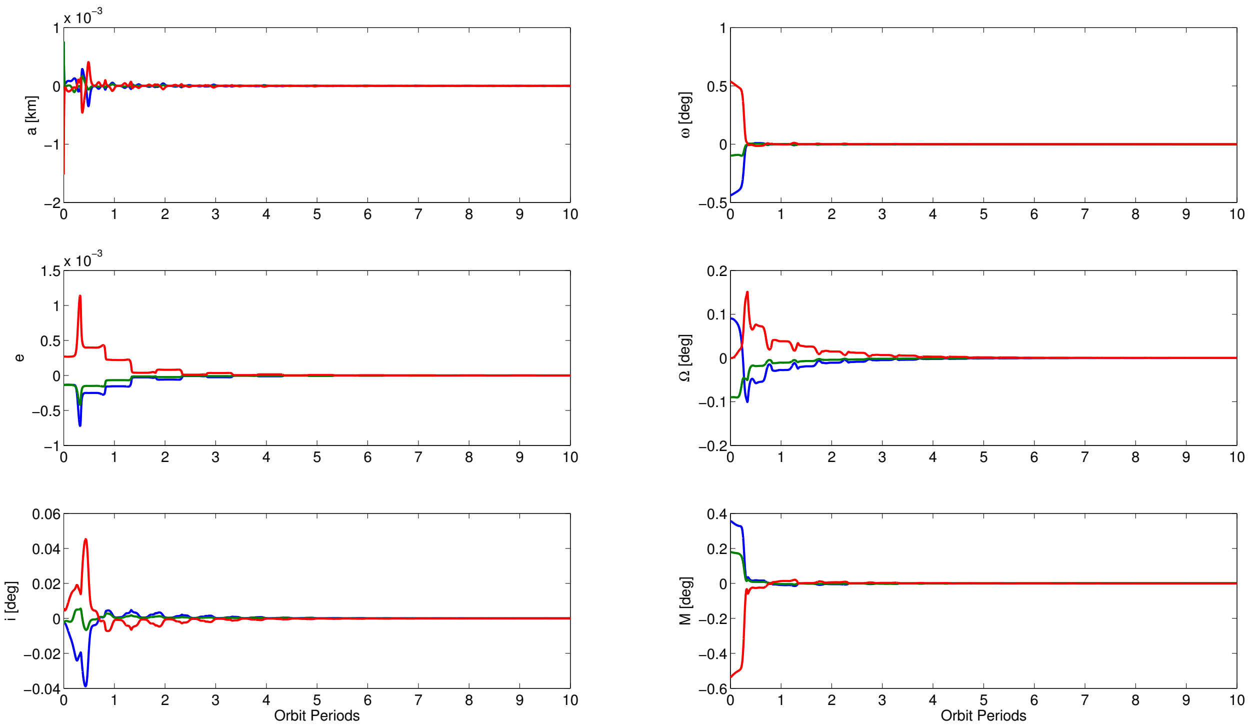



Figure 4: Mean orbit element errors versus time for each spacecraft.

barycenter to their desired formation slots. One worry during this process is the range between spacecraft as we want to avoid close encounters as much as possible. The relative ranges between spacecraft are shown in Fig. 6. There is a fairly close pass near the start of this simulation, which is a result of the initial conditions being close to one another. However the controller quickly recovers and pushes the spacecraft apart to safer orbits near the designed formation.

Fig. 7 shows the control magnitude used by each spacecraft. The peak magnitudes used by this controller is on the order of 1 cm/s$^2$. We note, however, that no attempt was made to minimize the amount of control usage here. The controller is a continuous controller, and keeps executing even when the mean element errors become very small.

Finally, Fig. 8 shows the differences between the weighted orbit element barycenter and the designed barycenter. Recall that this controller makes no attempt to null these differences. This results in nearly constant offsets between the two barycenters in $a$, $e$ and $i$ once the formation settles into the desired slots. The other three elements show a secular change in the difference; this is due to the fact that because the barycenters are different in $a$, $e$ and $i$, this causes different secular rates due to $J_2$.

### B. Barycenter Control with Perfect Navigation

The second test case is exactly like the previous case, except that the barycenter controller (Section III.B) is used instead of the formation controller. This means that the weighted barycenter is driven to the designed barycenter.

The relative performance of the spacecraft in this simulation are very similar to those presented in Figs. 3 - 6, and therefore these plots are not reproduced here for the sake of brevity.

The main differences between this case and the previous case are encompassed in Figs. 9 and 10. First, we see that in Fig. 10, the difference between the weighted barycenter and the designed barycenter are driven to zero for all orbital elements. However, this comes at a cost; comparing the control magnitudes in Fig. 9

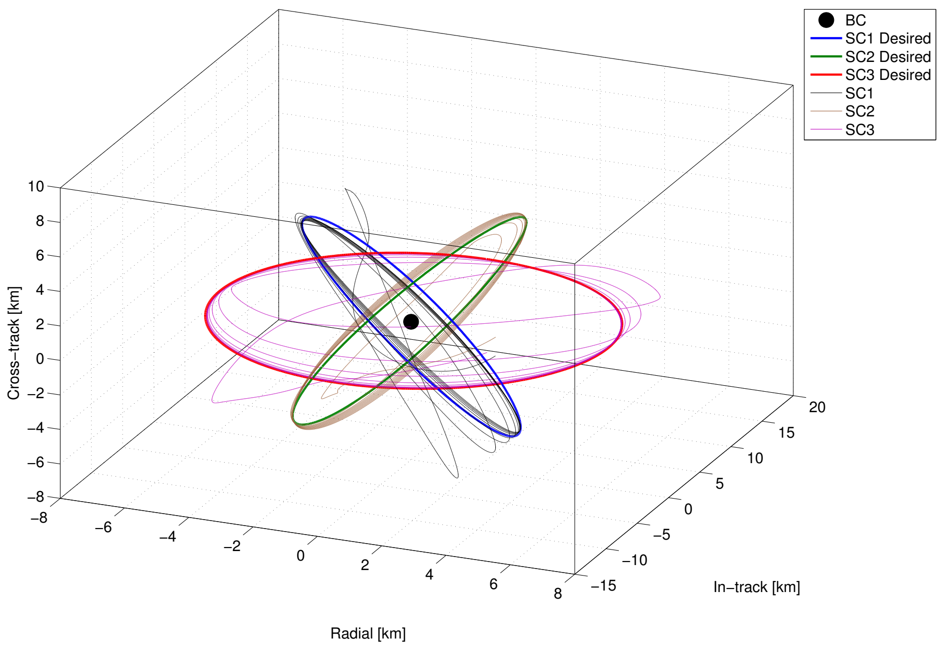



Figure 5: The Hill frame trajectories of the spacecraft relative to the weighted barycenter. Each spacecraft approaches the desired formation slot.

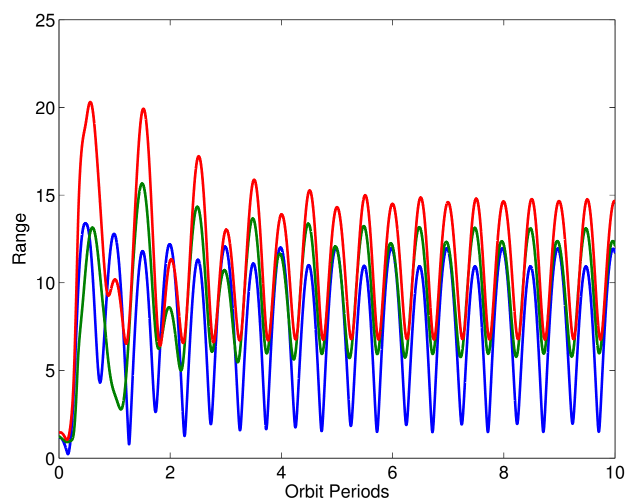



Figure 6: Physical distances between the spacecraft, using the same color scheme as in Fig. 2(b).

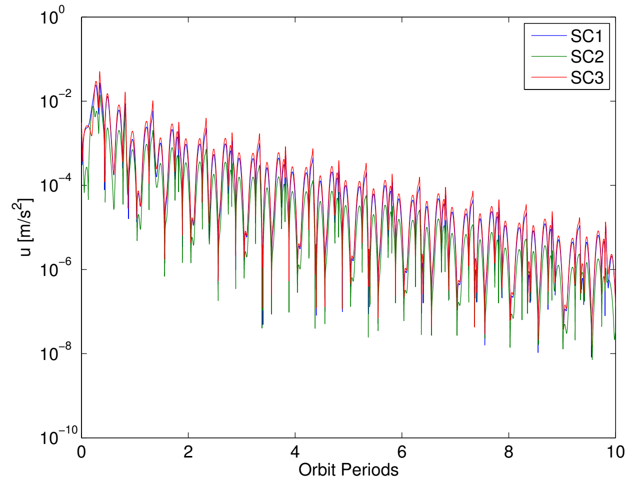



Figure 7: Control magnitude for each spacecraft in log-scale.

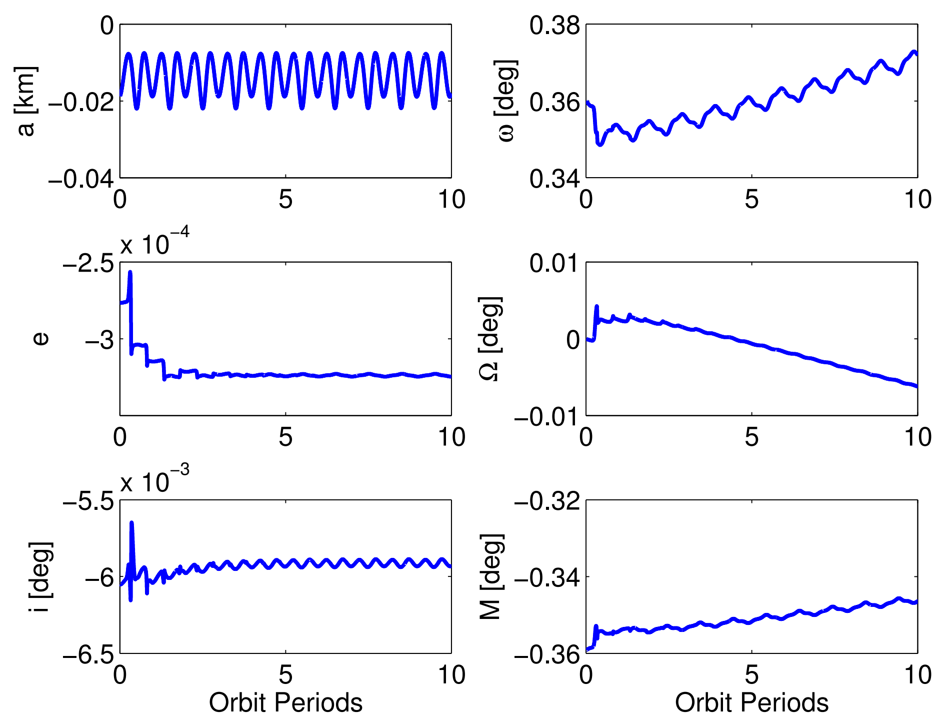



Figure 8: Mean orbit element differences between the actual weighted barycenter and the designed barycenter given in Eq. (29).

to those from the previous case in Fig. 7 we see that instead of having the controls decrease exponentially towards zero, some constant level of control is required by the barycenter controller in order to ensure the barycenter does not drift from the designed barycenter. This is a consequence of the inability to design a perfectly $J_2$ invariant formation with respect to the weighted barycenter, as outlined in Section II.

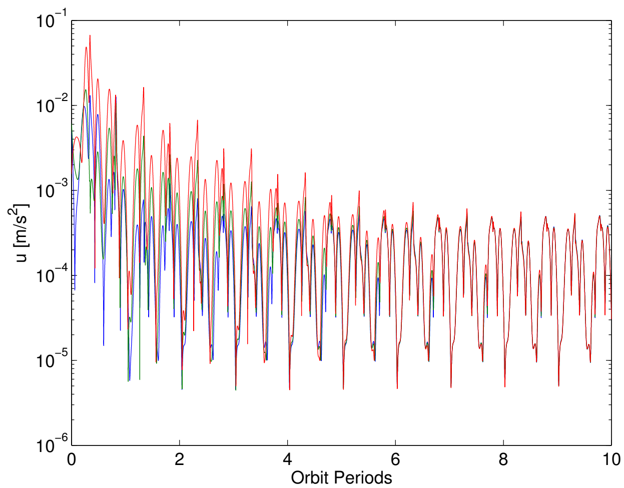



Figure 9: Control magnitude for each spacecraft in log-scale.

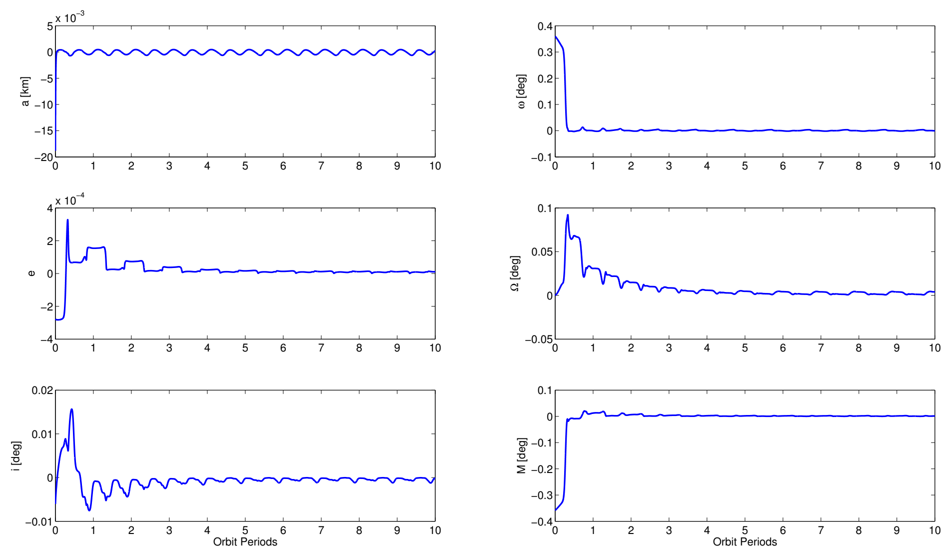



Figure 10: Mean orbit element differences between the actual weighted barycenter and the designed barycenter given in Eq. (29).

### C. Formation Control with Consensus Navigation

This case is similar to the first case in that the formation controller is tested. However we now add the consensus navigation routine from Part I[1] so the controller for each spacecraft uses its own estimate of the weighted barycenter and its mean orbit elements.

The main difference in the implementation of the formation controller here compared to with perfect navigation knowledge is the rate at which the controller is executed, as discussed in Section C. In this simulation, the filter is run at 1 Hz, while the controller is run at 0.04 Hz. This factor of 25 difference allows the filter to converge after each control execution before a new control thrust is executed. The control gains were also reduced to 75% of the previous values. We also allow for an uncontrolled portion at the start of the trajectory; for approximately the first tenth of an orbit, the controller is disabled to allow the filters to converge. For these simple simulations, this is equivalent to preventing the controller from executing when large filter errors are present as discussed in Section C.

The initial conditions used for the spacecraft mean orbit elements are given in Table 2. The initial conditions used in the filters were randomized according to the a priori filter covariances. Since each spacecraft keeps its own estimate of each of the other spacecraft, this means that there was an initial error in the location of the other spacecraft, as well as itself, in each filter. This leads to an error in the weighted barycenter estimate as well.

It should also be noted here that the filter uses its state knowledge to determine the local vertical - local horizontal frame in which the controller computed thrust is applied. Therefore, when the filter has any error from truth, it is applying the thrust incorrectly in the ECI frame where the estimation is occurring.

Table 2: Initial Conditions for nominal Simulations

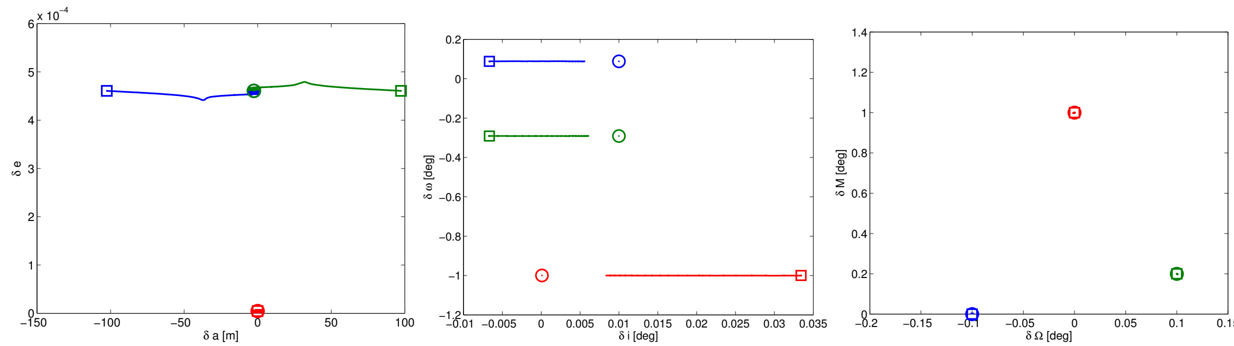
| $\delta\bar{\mathbf{e}}_0^i$ | SC1 | SC2 | SC3 |
|---|---|---|---|
| $\delta a$ | $\delta\bar{a}_{r,0}^1 - 0.1$ | $\delta\bar{a}_{r,0}^2 + 0.1$ | $\delta\bar{a}_{r,0}^3$ |
| $\delta e$ | $\delta\bar{e}_{r,0}^1$ | $\delta\bar{e}_{r,0}^2$ | $\delta\bar{e}_{r,0}^3$ |
| $\delta i$ | $\delta\bar{i}_{r,0}^1$ | $\delta\bar{i}_{r,0}^2$ | $\delta\bar{i}_{r,0}^3 + 0.05°$ |
| $\delta\omega$ | $\delta\bar{\omega}_{r,0}^1$ | $\delta\bar{\omega}_{r,0}^2$ | $\delta\bar{\omega}_{r,0}^3$ |
| $\delta\Omega$ | $\delta\bar{\Omega}_{r,0}^1$ | $\delta\bar{\Omega}_{r,0}^2$ | $\delta\bar{\Omega}_{r,0}^3$ |
| $\delta M$ | $\delta\bar{M}_{r,0}^1$ | $\delta\bar{M}_{r,0}^2$ | $\delta\bar{M}_{r,0}^3$ |

The results are presented in the same order as the first case. Fig. 11 shows the mean orbit element trajectories, while Fig. 12 shows the mean orbit element errors for each spacecraft with respect to the desired formation slot. The controller is moving each spacecraft toward the desired formation slots, however the convergence is much slower and noisier than in the case with perfect navigation knowledge. For example, note that due to the decreased efficiency of the controller in this case, 10 orbits was not long enough to get the spacecraft all the way to their formation slots, especially in inclination. Fig 13 shows the trajectories in the Hill frame with respect to the weighted mean barycenter as they converge to the desired relative orbits. Fig. 14(a) verifies that there are no collisions between spacecraft as they converge to the desired formation.

Fig. 14(b) shows markedly different control performance than in the cases with perfect navigation. The control magnitudes are much higher in this case, mainly due to the fact that the controller execution frequency is much slower. The same trend appears as before, however, in that the control magnitude is decreasing over time as the formation converges to the desired slots.

Fig. 15 shows the differences between the actual weighted barycenter and the designed barycenter. As with the previous case using the formation controller, there are roughly constant errors in $a$, $e$, and $i$, while the errors in the other three orbit elements are changing secularly due to the lack of $J_2$ invariance.

The filter estimates of the weighted barycenter for each spacecraft are shown in Fig. 16. The initial filter errors are quickly damped out and the errors go toward zero. Due to the nature of the estimation problem, however, the estimates are noisy based on the noise in the measurements themselves. This noise is a major factor in the slow, noisy convergence to the desired formation slots seen in Fig. 12.




(a) $\delta a$ - $\delta e$ (b) $\delta i$ - $\delta\omega$ (c) $\delta\Omega$ - $\delta M$

Figure 11: Spacecraft trajectories in differential mean orbit element space, shown as three 2-dimensional subspaces. The squares signify the starting position, and the circles are the targeted formation slots. Note that SC1 and SC2 have the same starting and targeted locations in the $\delta a - \delta e$ subspace.

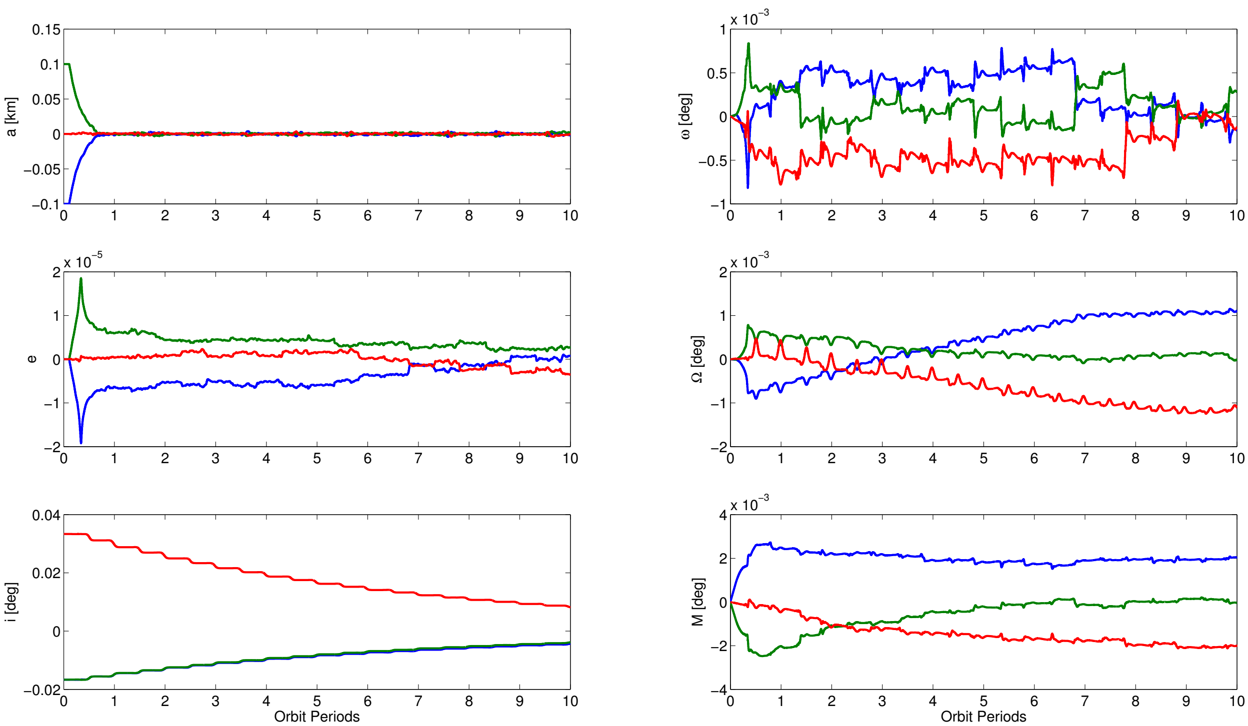



Figure 12: Mean orbit element errors versus time for each spacecraft.

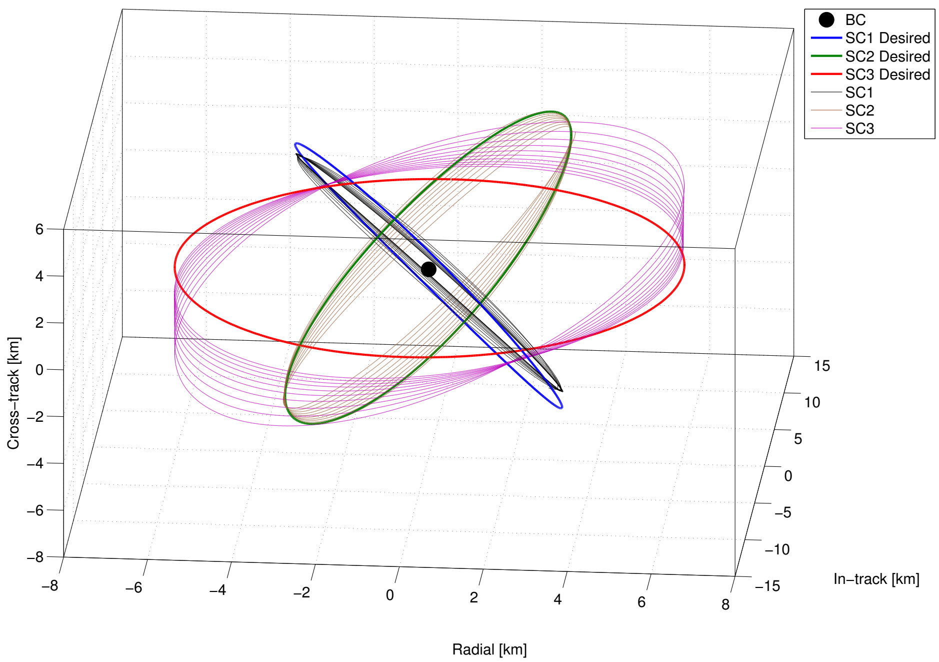
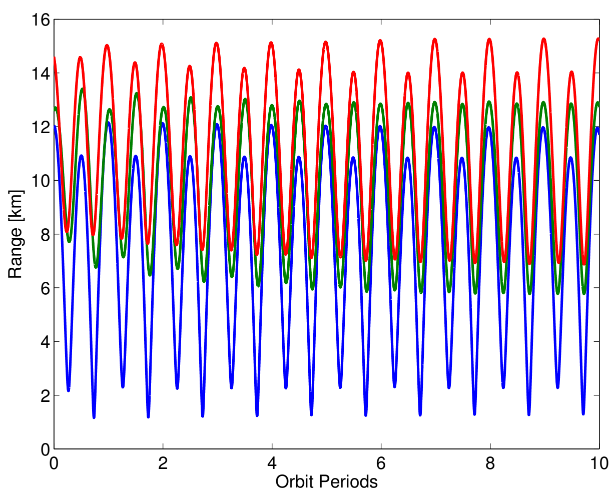
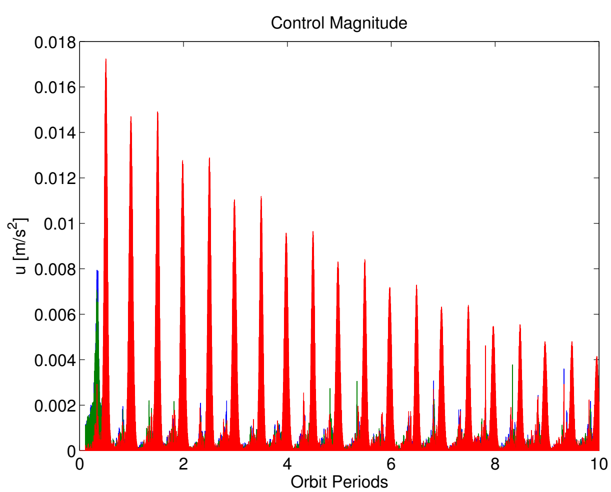



Figure 13: The Hill frame trajectories of the spacecraft relative to the weighted barycenter. Each spacecraft approaches the desired formation slot.




(a) Ranges




(b) Controls

Figure 14: Physical distances between the spacecraft, using the same color scheme as in Fig. 2(b) and control magnitude for each spacecraft.

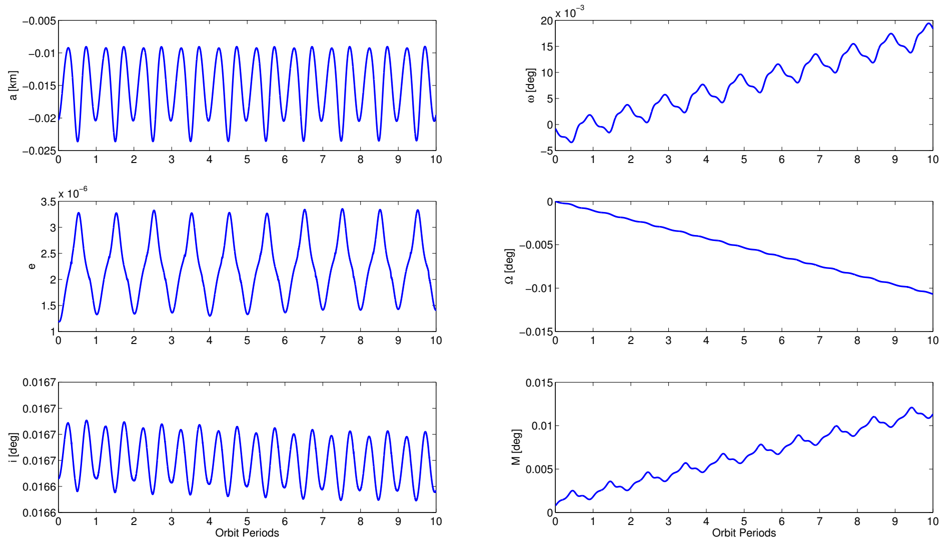



Figure 15: Mean orbit element differences between the actual weighted barycenter and the designed barycenter given in Eq. (29).

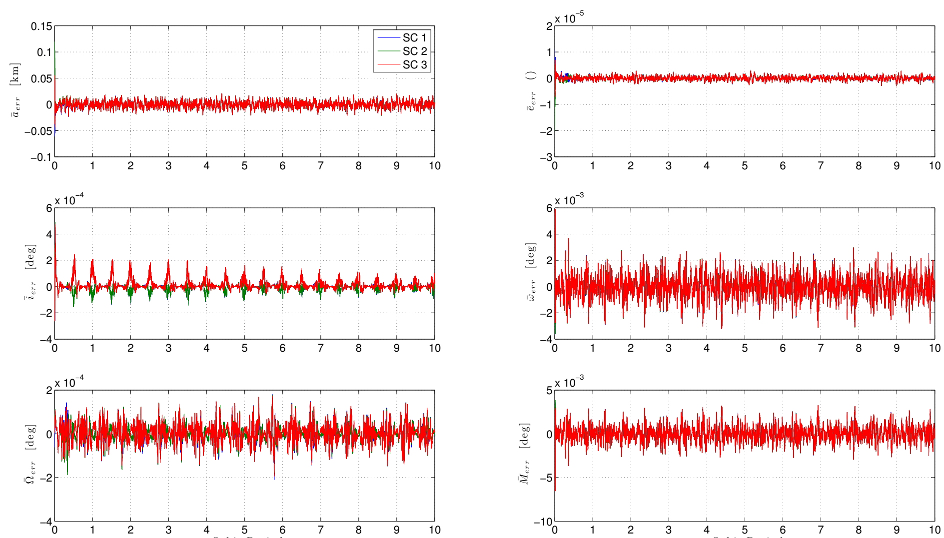



Figure 16: The error between each spacecraft's estimate of the weighted barycenter, and the truth position, in mean orbit elements.

### D. Barycenter Control with Consensus Navigation

This test case is exactly like the previous case, except that the barycenter controller (Section III.B) is used instead of the formation controller. This means that the weighted barycenter is driven to the designed barycenter. The filter initial conditions are different here, but again these errors are quickly nulled and don't have a large effect on the performance.

The relative performance of the spacecraft in this simulation are very similar to those presented in Figs. 11 - 14 and 16, and therefore these plots are not reproduced here for the sake of brevity.

The main differences between this case and the previous case are encompassed in Figs. 17 and 18. First, we see that in Fig. 18, the controller is trying to null out the difference between the weighted barycenter and the designed barycenter for all orbital elements. However, this comes at a cost; comparing the control magnitudes in Fig. 17 to those from the previous case in Fig. **??** we see that the control magnitude is significantly higher here. In this case, the controller is attempting to fight the secular change in $\omega$, $\Omega$, and $M$, which in turn cases some errors in $e$ that weren't seen before. However note that instead of roughly constant errors in $a$ and $i$, these are being driven to zero.

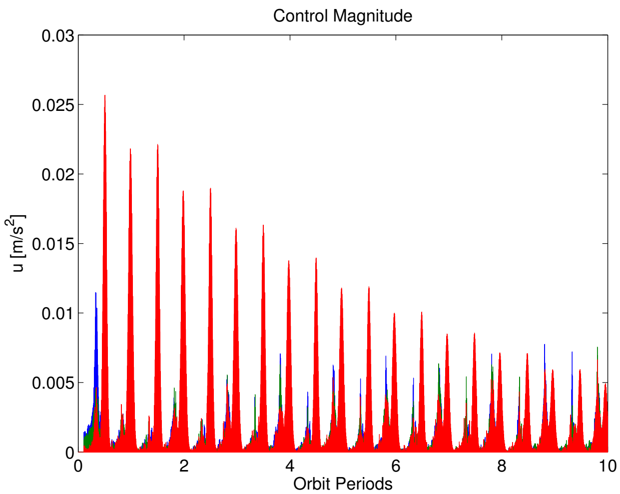



Figure 17: Control magnitude for each spacecraft.

## V. Conclusions

This paper presented several methods for formation flight control around a weighted mean orbit element barycenter. The methods were tested both with and without the consensus algorithm, and were showed to be able to stabilize the formation as desired. However, the performance was noticeably degraded once the consensus was used to provide navigation. The paper also presented a number of results on desirable spacecraft weightings, modifications for the presented controllers, and formation design constraints. One of the key results was to show that for an arbitrary formation, $J_2$ invariance can not always be satisfied, except for the special case with a chief spacecraft at the barycenter of the formation. In light of this, actually implementing a precisely $J_2$ invariant orbit may be more difficult than previously thought.

There is a wide variety of extensions to the project that can be carried out for future work. First, the modifications to the controllers suggested (deadband, adaptive formation slots, covariance based conrol) in this paper should all be implemented and tested. Furthermore, the results found here should be compared with other controllers from the literature. Most importantly, the relationship between the controllers and the consensus algorithm needs to be understood more completely. This information may allow for a new controller design that will perform better in the presence of consensus errors.

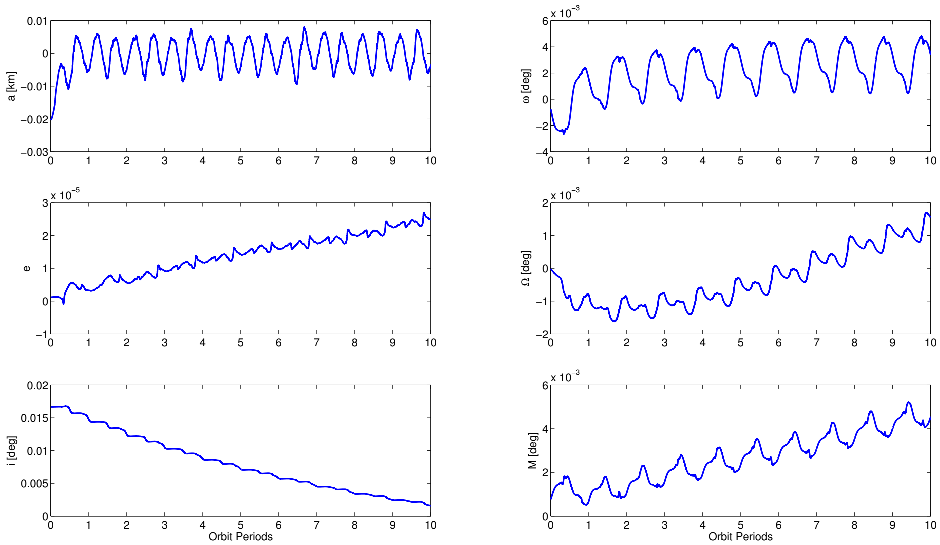



Figure 18: Mean orbit element differences between the actual weighted barycenter and the designed barycenter given in Eq. (29).

## Acknowledgments

The authors would like to thank Daniel Scheeres, Eric Frew, and Hanspeter Schaub for excellent introductions to the core material used in this paper. We would also like to acknowledge Mark Milam and Jeremy Schwartz for discussions of their thoughts on related topics.

## References

[1]Holzinger, M., and McMahon, J., "Decentralized Mean Orbit-Element Formation Guidance, Navigation, and Control: Part 1," 2012 Astrodynamics Specialist Conference, American Institute of Aeronautics and Astronautics, Reston, VA (submitted for publication)

[2]Schaub, H. P. and Junkins, J. L., Analytical Mechanics of Space Systems, AIAA Education Series, AIAA, Inc., Reston, VA, 2003.

[3]Schaub, H., Vadali, S., Junkins, J., and Alfriend, K., "Spacecraft Formation Flying Control using Mean Orbit Elements," *Journal of the Astronautical Sciences*, Vol. 48, No. 1, 2000, pp. 69-87

[4]Stengel, R. F., Optimal Control and Estimation, Dover Publications Inc., New York, NY, 1994

[5]Schaub, H., and Alfriend, K., "Hybrid Cartesian and Orbit Elements Feedback Law for Formation Flying Spacecraft, *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 2, MarchApril 2002, pp. 387393.

[6]Schaub, H. and Alfriend, K. T., "Impulsive Feedback Control to Establish Specific Mean Orbit Elements of Spacecraft Formations," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 4, 2001, pp. 739-745.