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DESIGN SPACE PRUNING HEURISTICS AND GLOBAL OPTIMIZATION METHOD FOR CONCEPTUAL DESIGN OF LOW-THRUST ASTEROID TOUR MISSIONS

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ABSTRACT

Electric propulsion has recently become a viable technology for spacecraft, enabling shorter flight times, fewer required planetary gravity assists, larger payload masses, and/or smaller launch vehicles. With the maturation of this technology, however, comes a new set of challenges in the area of trajectory design. In 2006, the 2nd Global Trajectory Optimization Competition (GTOC2) posed a difficult mission design problem: to design the best possible low-thrust trajectory, in terms of final mass and total mission time, that would rendezvous with one asteroid in each of four pre-defined groups. Even with recent advances in low-thrust trajectory optimization, a full enumeration of this problem was not possible. This work presents a two-step methodology for determining the optimum solution to a low-thrust, combinatorial asteroid rendezvous problem. First is a pruning step that uses a heuristic sequence to quickly reduce the size of the design space. Second, a multi-level genetic algorithm is combined with a low-thrust trajectory optimization method to locate the best solutions of the reduced design space. The proposed methodology is then validated by applying it to a problem with a known solution.

INTRODUCTION

With the recent launches of Deep Space 1[1], SMART-1[2], Hayabusa[3], and Dawn[4], electric propulsion has become a viable option for solar system exploration. Electric propulsion has the potential to result in shorter flight times, fewer required planetary gravity assists, and/or smaller launch vehicles[5]. One major challenge of low-thrust missions is in the area of trajectory design and optimization. At present, mission design often relies on local optimization of the low-thrust trajectories using expert-based starting points for departure and arrival dates and selection of gravitational swing-bys. These choices are generally based on known configurations that have worked well in previous analyses or simply on trial and error. At the conceptual-design level, however, exploring the full extent of the design space – over a large range of potential launch dates, flight times, and target bodies – is important in order to select the best possible solution or set of solutions. Global optimization is difficult because this design space is often multi-model and even discontinuous when considering multiple targets. Furthermore, solving a single low-thrust trajectory optimization problem for a given launch date and flight time is a non-trivial problem and can require a considerable amount of computing power and user oversight.

In 2006, the 2nd Global Trajectory Optimization Competition (GTOC2)[6,7] posed a trajectory optimization problem of a "Grand Asteroid Tour." This problem was chosen to be representative of the challenges mission designers face when designing low-thrust trajectories to multiple bodies in the solar system. Over the span of four weeks, 26 teams attempted to design the best possible trajectory, using electric propulsion, that would rendezvous with one asteroid from each of four defined groups. Only 15 of the 26 teams were able to submit solutions by the deadline, and only 11 of those solutions satisfied all of the problem constraints, indicating how difficult such a problem can be to solve.

The given objective function rewarded trajectories with low propellant consumption and low total flight time. Earth launch date, Earth launch v_∞, times of flight, and stay times at each asteroid were free design variables. Fig. 1 plots the set of asteroids for the GTOC2 problem, as a function of inclination, eccentricity, and semi-major axis. Group 4, which is comprised of asteroids closest to Earth, contained 338 asteroids. Group 3 has 300 asteroids, Group 2 has 176 asteroids, and Group 1, whose asteroids are the furthest from Earth, has 96 asteroids.

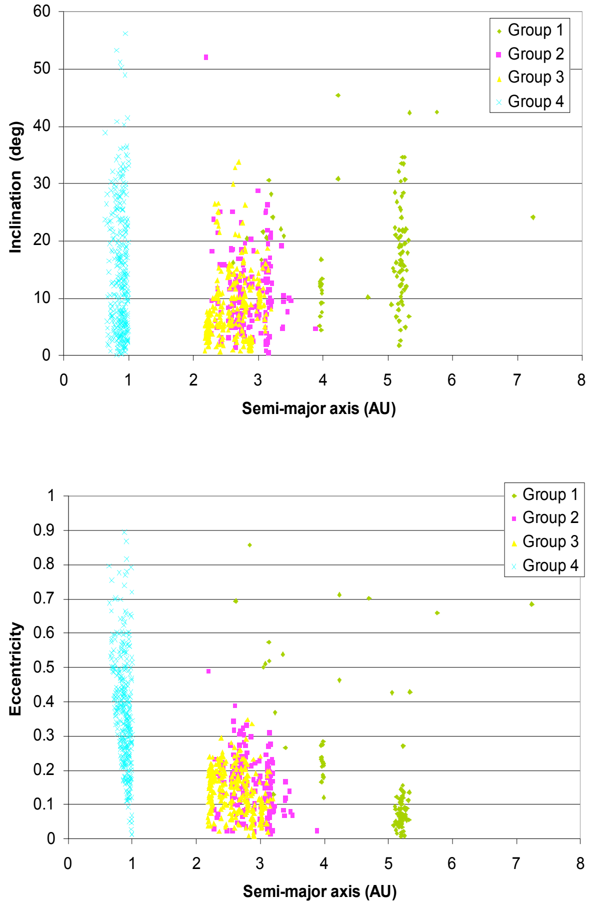

Fig. 1: GTOC2 set of asteroids.

As such, this problem permits 41 billion possible discrete asteroid combinations. The size of this problem is increased further when launch date, times of flight, and stay times are included as free design variables. Table 1 presents the remaining constraints that were placed on the problem. There is no constraint on the direction of the hyperbolic excess velocity (V_∞) at Earth launch, the thruster can be turned on and off at will, and there is no constraint on the thrust direction. Furthermore, no gravity assists were permitted in the competition.

Constraint	Value
Earth Launch V_∞	$\leq$ 3.5 km/s
Earth Launch Date	2015 – 2035
Asteroid Stay Time	$\geq$ 90 days
Total Flight Time	$\leq$ 20 years
Spacecraft Initial Mass	1500 kg
Spacecraft Propellant Mass	1000 kg
Thruster Isp	4000 s
Maximum Thrust	0.1 N

Table 1: Additional constraints on the GTOC2 problem.

The GTOC2 problem is a global optimization problem over a large design space with numerous local optima, for which an already existing method or software would not suffice. The large number of possible asteroid combinations prohibited each and every one from being examined, particularly because of the large launch range on launch date and total flight time. Furthermore, the phasing aspect of the problem leads to a multi-modal design space, which prevents a standard gradient-based optimizer from being used on a particular asteroid combination. Even if the optimal solution for a single asteroid combination could be obtained in one second, examining all 41 billion combinations would take 1303 years! In order to evaluate every combination in the allotted 4-week time frame, the optimal solution for each asteroid combination would have to be obtained in less than 6×10^{-5} seconds.

In order to make the problem more manageable, all of the participating teams first employed some form of pruning step in order to eliminate what they believed to be the worst solutions from the design space. This step included removing both asteroid combinations and portions of the launch date and flight time domain for particular asteroid combinations. Most teams cited one of their major weaknesses to be their chosen pruning technique, believing that they actually eliminated some of the best solutions from the design space. The chosen pruning techniques fell into several categories: ephemeris-based metrics that eliminated specific asteroid combinations, phase-free approximations that also eliminated asteroid combinations, and metrics that took phasing into account, thereby eliminating areas of the time domain. The teams then used their particular low-thrust optimization tools to optimize the most promising asteroid combinations and date ranges obtained from the pruning step.

This paper presents a methodology for determining the best set of solutions for a combinatorial asteroid rendezvous problem, motivated by the GTOC2 problem. The proposed methodology consists of two steps, the first which quickly eliminates bad solutions from the design space, and the second which then locates the best solutions from the reduced design space using a global optimization method. The proposed methodology is validated by applying it to a smaller asteroid rendezvous problem, for which the optimal solution is known.

PROPOSED METHODOLOGY

Phase 1: Pruning

The goal of the pruning step is to quickly reduce the size of the problem by several orders of magnitude. This is accomplished using various heuristics specific to the physics of the underlying problem, in order to identify areas of the design space that will likely yield poor solutions in terms of the objective function. Heuristic methods, however, can not guarantee that only bad solutions will be eliminated from the design space. The goal of this first phase is to ensure that a large percentage of the best solutions remain for the second phase of the methodology.

A number of heuristics are available that can be used to predict the performance of the low-thrust trajectory without having to actually perform low-thrust trajectory optimization. The first type of pruning methods considered was ephemeris-based. These methods require almost no computation time to implement as they simply involve manipulating the given asteroid ephemeris data. These methods include the absolute value and/or change in semi-major axis, eccentricity, inclination, longitude of the ascending node, and orbital energy. As an example, large inclination changes generally result in large propellant expenditures. Therefore, asteroids combinations could be eliminated based on the magnitude of the inclination difference between two asteroids.

Another potential pruning method would be to solve for the two-impulse transfer between two asteroids. These solutions can either take phasing into account or be phase-free. A phase-free solution would determine the minimum ΔV required to transfer from one orbit to another, over a range of departure and arrival true anomalies. If phasing is taken into account, then the actual position of the asteroids is used for a given departure date and flight time. Lambert's problem can then be solved to determine the ΔV required for that particular transfer. Phasing could also be taken into account by considering either the distance or phase angle between two orbits, for a particular date and flight time.

The final methodology includes three pruning heuristics: (1) semi-major axis, (2) angle between the angular momentum vectors, and (3) optimum two-impulse, phase-free ΔV. More information on the selection process of these three metrics can be found in Ref. 8. Basically, these metrics were chosen because they had the highest correlation between the metric value and the final mass obtained from the low-thrust trajectory optimization for a set of test cases.

The first metric, semi-major axis, eliminates all asteroids combinations where the semi-major axis does not increase from asteroid to asteroid. If final mass is the only consideration, it makes intuitive sense to visit the asteroids in order of increasing or decreasing distance from Earth. When time of flight is also part of the objective function, it is necessary to visit the asteroids in order of increasing distance from Earth to reduce the overall flight time of the mission. Semi-major axis is therefore the easiest metric to use to address this consideration. The second metric involves eliminating asteroid combinations based on the angle between their angular momentum vectors. This angle can be calculated as follows, where h_i is the angular momentum vector of asteroid i:

$$\cos(\theta_h) = \frac{\vec{h}_i \cdot \vec{h}_{i+1}}{\|\vec{h}_i\| \cdot \|\vec{h}_{i+1}\|} \qquad [1]$$

This is used in place of inclination change between asteroids, as it results in a higher correlation with the low-thrust final mass. This higher correlation is due to the fact that inclination change alone does not take into account the relative orientation of the two orbits. The final pruning metric eliminates asteroid combinations based on the associated optimal, phase-free, two-impulse ΔV. To calculate this ΔV for a particular asteroid combination, the true anomaly at departure and arrival were discretized between 0 and 2π. Each possible combination of departure true anomaly and arrival true anomaly defines r1, r2, and the transfer angle, from which the minimum ΔV can be calculated over all possible revolutions and flight times[9]. Of course, there is no guarantee that the optimal asteroid configuration for a given asteroid pairing will occur during the date range of a particular problem, but the idea behind this technique is to identify the most "reachable" asteroids.

The chosen heuristic metrics are applied sequentially, based on computation time. In this way, the size of the problem will already have been reduced when the more time-intensive pruning metrics are applied. First, increasing semi-major axis is applied across all possible asteroid combinations. This will reduce the number of combinations by a factor of $n!$, where n is the number of asteroid groups to be visited. Next, the angle between the angular momentum vectors, θ_h, is applied on a leg-by-leg basis. Therefore, this metric is first applied to the Earth – Asteroid 1 leg, eliminating the worst k% of asteroid combinations based on this angle. It is then applied to the Asteroid 1 – Asteroid 2 leg, and so forth, up to the Asteroid n-1 – Asteroid n leg. The percent of asteroid combinations eliminated is chosen based on the desired reduction in the size of the problem. Of course, the larger the percentage eliminated, the greater the probability is of eliminating some of the best combinations from the design space. Finally, the optimal, phase-free, two impulse ΔV is applied, also on a leg-by-leg basis, to the remaining asteroid combinations.

Phase 2: Global Optimization

In the second phase, a global optimization algorithm is applied to the reduced design space to locate the optimal solution. The global optimizer is responsible for the following design variables: asteroid combination, launch date, times of flight, and stay times. This system-level optimization is coupled with a local low-thrust trajectory optimization scheme that determines the optimal control history of the spacecraft in order to minimize propellant for a given set of global optimization variables.

A genetic algorithm (GA) is chosen as the global optimization algorithm. A genetic algorithm is a domain-spanning, probabilistic optimization algorithm based on the Darwinian theory of evolution. Although there are numerous variations, the general genetic algorithm begins with a random initial population, which is made up of multiple sets of values for each of the design variables. Each member of the population represents a single value for each of the design variables. This generally results in a random scatter of points over the design space. Each set of design variables is referred to as a chromosome and is typically encoded as a binary string, which must be mapped to the real values of the variables. The design variables are discretized between their lower and upper bounds. In each generation, the population undergoes certain genetic operators such that the population will "evolve" and improve its fitness (objective function). The typical genetic operators are reproduction, crossover, and mutation. The purpose of reproduction is to weed out the members of the population with low fitness, and to keep those with high fitness. Crossover combines two "parents" by switching parts of their chromosome strings with each other, while mutation is responsible for switching individual bits in a chromosome string. Because there is no necessary condition for optimality, the convergence criteria is usually chosen either as a maximum number of generations (iterations) or a certain number of generations with no change in the objective function. As the generations progress, there should be a steady improvement in the both the average fitness of the population as well as the fitness of the best member. In general, at the termination of the GA, the population will be clustered around the global optimum[10,11].

One of the main advantages of genetic algorithms is their ability to find a global optimum in a discrete, multi-modal design space. They can also handle a large number of variables, and require no initial guesses for the design variables. Genetic algorithms, however, do have some downfalls. Because of the probabilistic nature of the algorithm, there is no guarantee that the optimal solution will be found. Therefore, the GA must generally be run more than once to ensure optimality. Genetic algorithms also require a large number of iterations, and therefore a large number of function calls, in comparison to a gradient-based method. Finally, if the original design space is comprised of continuous design variables, the discretized solution will generally not correspond to the precise global optimum. A common practice is to use the solution obtained by the GA as an initial guess to a gradient-based optimizer, in order to improve the accuracy of the solution.

For this particular methodology, a two-level optimization is employed. The outer loop uses a genetic algorithm to solve for the asteroid combination. The inner loop also employs a genetic algorithm, this time to solve for the launch date, times of flight, and stay times for a given asteroid combination. This configuration had better performance in test cases than combining all of the design variables into one genetic algorithm. When combining all of the design variables, two methods were tried: (1) the asteroid combination was described using a single variable (combination #), and (2) the asteroid combination was described using three variables (asteroid 1, asteroid 2, asteroid 3). For the two-level setup, each genetic algorithm uses binary encoding for the design variables. Tournament selection is used for reproduction, with a tournament size of 4. Finally, two-point crossover and string-wise mutation are employed.

The inner loop is then coupled with a low-thrust trajectory optimization algorithm, in order to determine the optimal control history of the spacecraft for a given asteroid combination, launch date, times of flight, and stay times. MALTO is used to perform the local low-thrust trajectory optimization, which is a low-thrust trajectory optimization tool developed at JPL based on the direct method by Sims and Flanagan[12]. MALTO is a medium-fidelity tool, which provides good results in comparison to higher fidelity tools, but can be run in a much more automated fashion and generates results quickly.

VALIDATION

Validation Problem

In order to evaluate the effectiveness of the proposed methodology, a small validation problem with a known solution was developed. A subset of the GTOC2 problem was chosen, using many of the same constraints posed in the original problem but with a smaller set of asteroids. The validation problem contains 8 Group 1 asteroids, 8 Group 2/3 asteroids (combined due to their similar semi-major axis values), and 8 Group 4 asteroids, leading to 3,072 discrete asteroid combinations. Fig. 2 plots these asteroids, as a function of their semi-major axis, eccentricity, and inclination.

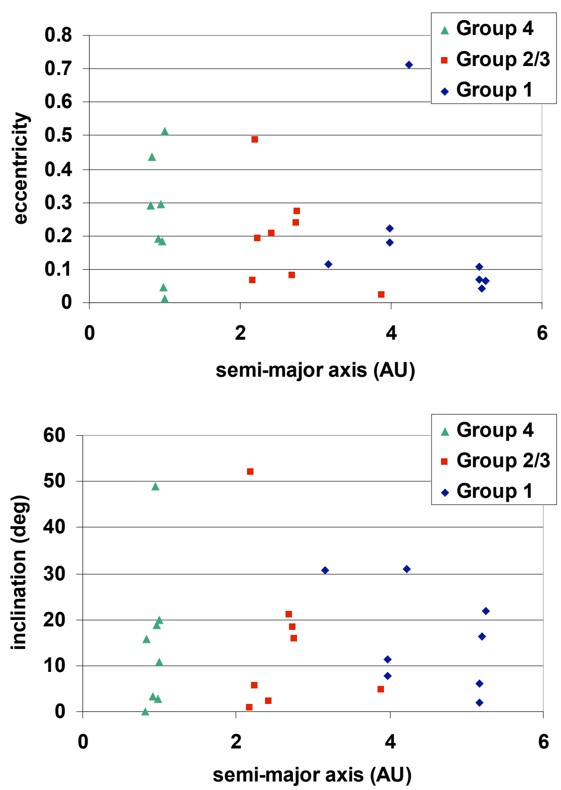

Fig. 2: Set of asteroids for sample problem.

The objective function for the validation problem is to maximize the final mass of the spacecraft. The following constraints were placed on the flight times: Leg1 $\leq$ 600 days, Leg 2 $\leq$ 1800 days, and Leg 3 $\leq$ 1200 days. These times were chosen based on visiting the asteroids in increasing order of semi-major axis. The validity of this assumption will be addressed in the subsequent section. Lastly, the launch window was shortened to fall between 2015 and 2025, inclusive, and the stay time at each asteroid was fixed at 90 days. While flight time no longer directly appears in the objective function, it is dealt with implicitly in the chosen constraints. The other assumptions laid out in GTOC2 were not changed in this sample problem. Launch from Earth is constrained by a hyperbolic excess velocity (V_∞) of up to 3.5 km/s with no constraint on direction. The spacecraft has a fixed initial mass of 1500 kg, which does not change with launch V_∞, and a minimum final mass of 500 kg. The propulsion is modeled to have a constant specific impulse of 4000 s and a maximum thrust level of 0.1 N, and can be turned on and off as needed.

Within the validation problem, MALTO was used to perform the local low-thrust trajectory optimization. A Fortran script was written that automatically generates the MALTO input file, runs MALTO, and then parses the output files for the relevant mass data. In order to solve the validation problem, the design space was discretized in terms of launch date and times of flight, and each leg of the trajectory was analyzed separately. The launch date from Earth was discretized in 30-day steps, and the time of flight to the first asteroid was discretized in 100-day steps up to the 600-day constraint. MALTO was used for each case to determine the departure V_∞ and thrust profile that maximizes the final mass at the arrival asteroid, based on a 1500 kg initial spacecraft mass. The time of flight for the second leg was also discretized in 100-day increments, up to 1800 days. For each feasible Leg 1 trajectory (final mass greater than 500 kg), the corresponding Leg 2 trajectory was calculated, for each of the discretized times of flight. Finally, the set of Leg 3 trajectories was calculated in a similar fashion, starting from all of the feasible Leg 2 trajectories. This approach allows not only the best asteroid combination to be determined, but the entire set of feasible solutions to be ranked by final mass.

The resulting set of feasible solutions contains 115 of the possible 512 asteroid combinations initially examined. This set of feasible solutions contains 4 Group 1 asteroids, 6 Group 2/3 asteroids, and all 8 Group 4 asteroids (although not every permutation of these 18 asteroids). The best solution to the discretized problem is plotted in Figure 3. The spacecraft departs Earth on March 1, 2015 with a launch V_∞ of 2.59 km/s. The time of flight for each leg is 600 days, 1600 days, and 1200 days, respectively. Interestingly, even though time of flight does not appear explicitly in the objective function, the flight time for the second leg is not equal to its upper bound. While an 1800-day time of flight would result in a larger final mass for that particular leg, the shorter flight time results in better phasing for the third leg, thereby maximizing the overall final mass of the trajectory. The total flight time from Earth departure to the final asteroid rendezvous is 3580 days, which includes the two 90-

day stay times at each intermediate asteroid, and the arrival mass is 903 kg.

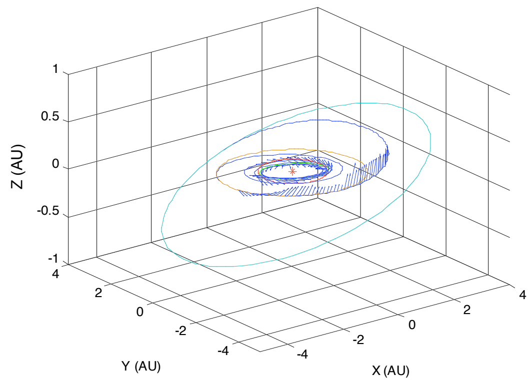

Fig. 3: Optimal solution for sample problem.

Table 2 lists the 10 best asteroid combinations, ordered in terms of final mass. Table 3 lists the Keplerian orbital elements of each of the asteroids that appear in Table 2, in the J2000 heliocentric ecliptic frame.

Phase 1: Pruning

The first step in the pruning process is to restrict the asteroid combinations to those with increasing semi-major axes. This immediately reduces the number of asteroid combinations in the validation problem from 3072 to 512. In order to determine if any feasible solutions were eliminated in this pruning step, the remaining possible asteroid combinations were analyzed, but without the individual leg time of flight constraints. The overall time of flight, however, is constrained to be no more than 3780 days (including the two 90-day stay times). Only two other feasible asteroid combinations were identified, both for the following order: Earth – Group 4 – Group 1 – Group 2/3. The maximum final mass for these two combinations was only 608 kg and 524 kg, which would rank 59th and 105th out of

Earth Dep. Date	Asteroid 1	Asteroid 2	Asteroid 3	Leg 1 TOF (days)	Leg 2 TOF (days)	Leg 3 TOF (days)	Final Mass (kg)
3/1/2015	"2006 QQ56"	Medusa	Kostinsky	600	1600	1200	904
8/12/2016	"2006 QQ56"	Hertha	Telamon	600	1800	1200	856
3/29/2021	Apophis	Hertha	Pandarus	300	1800	1200	843
1/1/2015	"2002 AA29"	Medusa	Kostinsky	600	1700	1200	831
9/11/2018	"2006 QQ56"	Geisha	Kostinsky	600	1700	1200	826
8/28/2015	"2006 QQ56"	Geisha	Caltech	600	1700	1200	812
3/1/3015	"2004 FH"	Medusa	Kostinsky	500	1800	1200	807
9/6/2019	"2006 QQ56"	Medusa	Potomac	600	1800	1200	804
7/18/2017	"2006 QQ56"	Geisha	Potomac	600	1800	1200	798
12/5/2019	Apophis	Medusa	Potomac	500	1800	1200	787

Table 2: Ten best asteroid combinations for validation problem, ranked by final mass.

Asteroid Name	Group	semi-major axis (AU)	eccentricity	inclination (deg)	longitude of the asc. node (deg)	argument of periapsis (deg)
"2002 AA29"	4	0.994	0.013	10.743	106.469	100.610
"2004 FH"	4	0.818	0.289	0.021	296.181	31.320
"2006 QQ56"	4	0.987	0.047	2.827	163.331	332.958
Apophis	4	0.922	0.191	3.331	204.460	126.396
Geisha	2/3	2.241	0.193	5.664	78.339	299.875
Hertha	2/3	2.428	0.207	2.306	343.898	340.035
Medusa	2/3	2.174	0.065	0.937	159.648	251.127
Caltech	1	3.162	0.114	30.690	84.608	294.922
Kostinsky	1	3.979	0.220	7.637	257.105	163.003
Pandarus	1	5.172	0.068	1.854	179.863	37.742
Potomac	1	3.980	0.181	11.402	137.512	332.820
Telamon	1	5.172	0.108	6.088	341.007	111.194

Table 3: Orbital elements of asteroids in Table 2, in the J2000 heliocentric ecliptic frame.

the now 117 feasible asteroid combinations. Therefore, this first pruning step was effective in keeping all of the best solutions in the design space for the validation problem.

The next two pruning steps apply the angle between the angular momentum vectors and the optimal, two-impulse, phase free ΔV sequentially to each leg of the trajectory. 30% of the asteroid combinations are eliminated for Leg 1, 25% for Leg 2, and 20% for Leg 3. The reducing percentage eliminated is due to the decreasing ability of each metric to act as a predictor of low-thrust mass for each subsequent trajectory leg. Because the initial mass for all Leg 1 trajectories is equal, the resulting mass is a function of mass fraction alone. For subsequent legs, however, the previous legs define the initial mass and departure date, so each asteroid combination is no longer directly comparable. Using the chosen percentages, these two steps combined further reduce the number of asteroid combinations from 512 to 143. 18 feasible combinations are eliminated from the design space, the best of which has a final mass of 653 kg, which ranks 37th among the feasible combinations. Therefore, once again, all of the best solutions were kept in the design space.

A larger percentage of combinations could be eliminated for a larger problem. Because of the small size of the validation problem, however, if 70% or greater of the combinations were eliminated in each step, there would be no remaining asteroid combinations. Furthermore, a larger percentage could be eliminated, if one were willing to remove some of the better combinations from the design space. Up to 65% could be eliminated in each step and the optimum solution would still remain in the design space. Most of the other best solutions, however, would be eliminated.

To further validate these two pruning steps, the correlation between each metric and the corresponding low-thrust final mass can be examined for each asteroid combination. Fig. 4 plots the maximum final mass for each asteroid pairing as a function of the angle between the asteroid's angular momentum vectors. For example, for all asteroid combinations containing the Leg 2 pairing Medusa – Potomac, the maximum final low-thrust mass is 804 kg. The correlation between these two metrics for each leg is -0.737, -0.557, and -0.643, respectively. Fig. 5 then plots the maximum final mass for each asteroid pairing as a function of the optimal, two-impulse, phase-free ΔV. Again, the correlation between these two metrics for each leg is -0.790, -0.592, and -0.529, respectively.

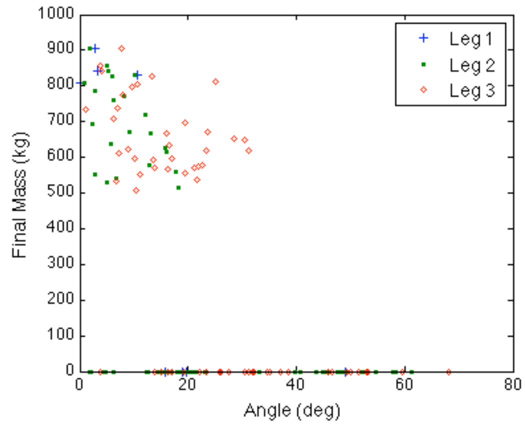

Fig. 4: Maximum low-thrust final mass as a function of the angle between the angular momentum vectors for each asteroid pairing.

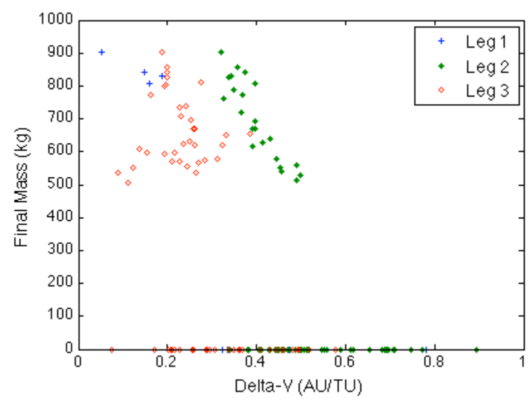

Fig. 5: Maximum low-thrust final mass as a function of the optimal, two-impulse, phase-free ΔV for each asteroid pairing.

Table 4 summarizes the results of the pruning phase as applied to the validation problem. The number of asteroid combinations was reduced from 3072 to 143, only 20 feasible combinations were eliminated, and the best asteroid combination eliminated ranked 37th. Therefore, the pruning step successfully reduced the size of the design space without eliminating the best solutions.

Pruning Metric	Trajectory Leg	% Combos Eliminated	# Combos Eliminated
$a_i < a_{i+1}$	All	N/A	2560
θ_h	Leg 1	30%	128
θ_h	Leg 2	25%	56
θ_h	Leg 3	20%	29
ΔV	Leg 1	30%	44
ΔV	Leg 2	25%	72
ΔV	Leg 3	20%	40

Table 4: Results of pruning phase applied to validation problem.

Phase 2: Global Optimization

As explained above, the global optimization phase is broken up into an outer optimization loop, responsible for calculating the optimal asteroid sequence, and an inner optimization loop, which determines the maximum final mass for a given asteroid sequence. The outer optimization loop has three design variables, one for each asteroid in the trajectory. These design variables are encoded as a binary string. Table 5 lists the settings used for the outer loop genetic algorithm. The initial population for the genetic algorithm is chosen randomly. The genetic algorithm is then run until a certain number of generations have elapsed with no change in the best ever fitness function. This value is chosen to be 25 for the outer optimization loop.

GA Setting	Value
Design variables	Ast.1, Ast.2, Ast.3
Population size	50
Stall generations	25
# Tournament participants	4
Probability of crossover	0.8
Probability of mutation	0.1

Table 5: Settings for the outer loop genetic algorithm.

For each setting of the outer loop design variables, the fitness function is calculating by calling the inner loop optimization, which is responsible for four design variables – Earth departure date, Leg1 time of flight, Leg 2 time of flight, and Leg 3 time of flight. Then for each setting of the inner loop design variables, MALTO is used to perform the low-thrust trajectory optimization to maximize final mass. Table 6 summarizes the settings used for the inner loop genetic algorithm.

GA Setting	Value
Design variables	Earth Dep. Date, TOF1, TOF2, TOF3
Population size	100
Stall generations	25
# Tournament participants	4
Probability of crossover	0.8
Probability of mutation	0.1

Table 6: Settings for the inner loop genetic algorithm.

In order to save on function evaluations, a system of archiving is used, such that the fitness function is recorded for each setting of the design variables. The fitness can then be obtained using a table look-up once that set of design variables has been calculated. This archiving is performed separately for both the outer and inner loop optimizations. The other advantage of this archiving system is that a number of the best overall solutions will also be recorded.

Fig. 6 plots the results of one run of the global optimization phase on the validation problem. Three values are plotted for each generation of the outer loop optimizer – the best current fitness function, the best ever fitness function, and the average fitness function (final mass of the spacecraft) over all members of the population. The optimum asteroid combination, dates, and times of flight were found successfully in this case. Furthermore, six of the best ten asteroids combinations were also identified, simply by using the archiving process, which requires additional memory but no additional computing time. The optimization terminated after 26 generations of the outer loop optimizer and required 111 function calls to the inner loop optimizer. On average, each run of the inner loop optimizer required 354 fitness function evaluations. This still represents a two-order of magnitude improvement over the number of MALTO runs required to solve the discretized problem.

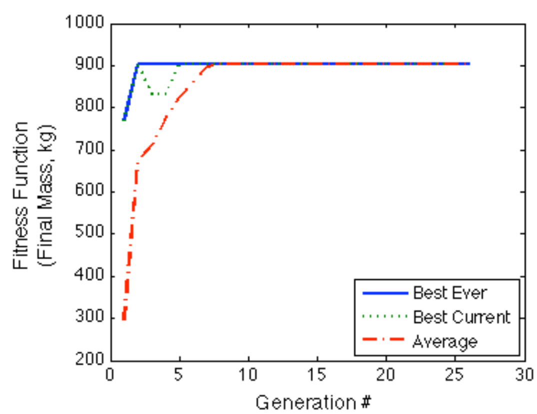

Fig. 6: Results of multi-level genetic algorithm applied to validation problem.

Because of the probabilistic nature of genetic algorithms, the results of each run are slightly different. Furthermore, there is no guarantee that the genetic algorithm will find the best solution. In running the multi-level genetic algorithm several times, however, it was found to have a greater than 50% percent success rate at locating the global optimum. Therefore, running it two or three times should be sufficient to find the global optimum.

CONCLUSIONS AND FUTURE WORK

This paper has presented a two-step methodology for solving a low-thrust, combinatorial asteroid rendezvous problem such as the problem posed in the 2nd Global Trajectory Optimization

Competition. The pruning step reduces the size of the design space by applying a set of heuristics to eliminate asteroid combinations with low values of the objective function. The global optimization step then uses a multi-level genetic algorithm to find the optimum of the reduced problem. Additionally, the use of archiving reduces the number of required function evaluations and records all of the solutions evaluated by the genetic algorithm. In conceptual design, it is often desirable to not only locate the single optimum solution, but also the best set of solutions to carry forward into the detailed design process.

While the methodology was successful in finding the global optimum of the validation problem, further work needs to be done to evaluate the effectiveness of the method on larger problems, approaching the size of the GTOC2 problem. For the validation problem, the pruning phase reduced the number of asteroid combinations from 3072 to 143, which represents a decrease of 95.3%. Of course, for a problem of this size, it is not possible to reduce the number of asteroid combinations by several orders of magnitude because there would be no remaining combinations in the design space. For a larger problem, however such a reduction would be necessary. It is expected that for a larger number of combinations, a higher percentage could be eliminated for each step of the pruning process, while still keeping the best combinations in the design space. The challenge of validating this theory, however, is in solving a larger combinatorial problem. For example, the GTOC2 problem is too large to discretize and solve, so there is no way to verify how close the solutions found during the competition were to the true global optimum, or how effective the team's pruning techniques were at eliminating only poor solutions.

The multi-level genetic algorithm was also successful in locating the global optimum of the validation problem. The multi-level setup required more overall MALTO function evaluations that using a single genetic algorithm responsible for all the design variables. The success rate of finding the global optimum, however, was much higher. While the multi-level GA had a better than 50% success rate, the single-level GA had less than a 10% success rate. The large number of MALTO function evaluations, however, could become prohibitive for a larger combinatorial problem. More work can be done on further optimizing the genetic algorithm settings to reduce the number of function evaluations. Additionally, combinatorial optimization methods, such as branch-and-bound type methods, could be evaluating as potential replacements for the outer loop genetic algorithm.

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