CubeSat Slosh Dynamics and Its Effect on Precision Attitude Control for the VISORS Distributed Spacecraft Telescope Mission

Christopher A. Carter[1] and E. Glenn Lightsey[2]

The VIrtual Super Optics Reconfigurable Swarm (VISORS) mission is a CubeSat distributed telescope mission which aims to take high angular resolution images of the sun using multiple spacecraft in sub 100m proximity operations. This requires high accuracy attitude and relative position control on small time scales. To achieve the required relative position accuracy, the spacecraft in the swarm must frequently use thrusters to provide impulsive corrections. This paper examines the effect that these maneuvers have on the CubeSat attitude control system by examining the relative magnitudes of each force acting on the system and choosing applicable analytic methods to predict the fundamental characteristics of the response. These characteristics were used to inform an analogous mechanical model for the slosh motion, which was used to determine the time-varying response of the spacecraft control system while considering slosh disturbances. By simulating typical maneuvers, the spacecraft's sensitivity to slosh disturbances was determined, providing operational constraints and initial validation of the mission's precise pointing requirements.

I. Introduction

As CubeSats become more capable, propulsion systems are being designed specifically for CubeSats. These propulsion systems give many new capabilities to CubeSats that were once restricted to larger satellites and missions such as deep-space momentum management, precise proximity operations, and orbital maneuvering. With propulsion systems, CubeSats are developing into a platform with capabilities comparable to traditional satellites. Combined with their relative low cost, maneuverability allows CubeSats to be used not only as technology development testbeds but also as science missions. An example of this is the recently created VIrtual Super Optics Reconfigurable Swarm (VISORS) mission, which aims to form a distributed telescope from two 6U CubeSats, proving multiple technologies while also promising images of the Sun at unprecedented resolutions.

To make the most efficient use of the available volume on a CubeSat, propulsion systems are being developed with liquid propellant, either as storage for cold-gas systems or directly as propellant. [1] With arcsecond level mission pointing requirements, the motion of these liquids and how it will interact with the spacecraft attitude control system becomes crucial to mission success. This paper examines the dynamics of the fluid within the VISORS spacecraft propulsion system and assesses the likely impact the propellant will have on the attitude control system.

This problem is addressed by first laying out the concept of operations of the propulsion system for the mission and identifying mission operations which have a noticeable effect on the propellant system. Then, experimentally derived relations are used to compute basic parameters such as natural frequency and damping ratio, which govern the time response. Using the defined characteristics of the unperturbed responses, analogous physical models are implemented to provide estimates of the responses to input disturbances, culminating in the evaluation of a simple proportional-derivative (PD) controller for the spacecraft attitude.

[1] Graduate Research Assistant, School of Aerospace Engineering, Georgia Institute of Technology ccarter73@gatech.edu

[2] Professor, School of Aerospace Engineering, Georgia Institute of Technology, glenn.lightsey@gatech.edu

There are two main approaches to modeling slosh dynamics. The first is to use computational fluid dynamics (CFD) to fully simulate the fluid motion. Direct CFD is considered to be the most accurate; however, combining CFD and spacecraft attitude dynamics requires significant computational time and can cause numerical instabilities. [2,3] The second approach is to use analogous mechanical models for the slosh dynamics. This method allows for rapid utilization in control and estimation algorithms, though it typically sacrifices accuracy and focuses only on the primary slosh mode and only on transverse slosh motion. [2,4] The most frequently used analogous models are a pendulum or a spring-mass-damper, with pendulum models being preferred for their ability to automatically adjust their natural frequency to the magnitude of gravitational accelerations.[5] The analogous models further differ in how they determine the mechanical parameters, with some using experimental curve fits and others using CFD. Each has advantages, though these methods provide static parameters, valid only when the fluid is under specific fluid regimes. [3,5] Thus, it is important, when modeling slosh dynamics, to understand the importance of the different forces acting on the fluid and to realize that the model is only valid in the fluid regime from which the parameters were derived. The effect of slosh dynamics on attitude control has been investigated by Veldman [3] and Souza [6]. Veldman combined CFD with attitude control to simulate the results of the Sloshsat FLEVO mission with a high degree of accuracy. Souza developed a mechanical model for the 2D attitude dynamics of a slosh-filled spacecraft. However, its model parameterization assumes a constant thrust, high-g acceleration, which is not applicable to low-thrust CubeSat missions such as VISORS. Further, it ignores capillary effects, which were found by Veldman to be important for microgravity analysis.[3] This paper differs from other approaches for modeling slosh in that it attempts to characterize the slosh response under microgravity conditions, with a focus on the fluid motion after propulsive or rotational maneuvers are performed. This is necessary due to VISORS' unique mission requirements, as the telescope mission's primary concern is achieving its precision pointing during science observations.

Concept of Operations for the VISORS spacecraft

The VISORS mission concept is a distributed telescope concept similar to the formation design specified by Koenig [7], with multiple spacecraft operating in close proximity to form the components of a distributed telescope. Each spacecraft is dedicated to an individual component. The main difference between the mission described by Koenig and VISORS is that VISORS targets the sun, rather than exoplanetary bodies. The specific target of this telescope are energy release sites in the sun's corona in the ultraviolet spectrum. The VISORS spacecraft will need to operate in close proximity, maneuvering within 20 meters of each other to form a distributed telescope. This formation concept is shown in Figure 1.

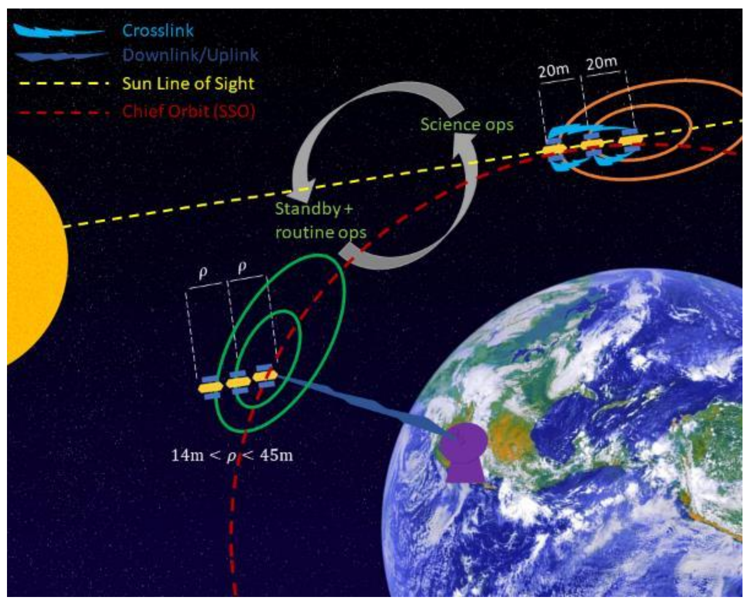

Figure 1: VISORS Formation Concept of Operations [8]

To image the sun with the desired angular resolution, the position and attitude of the spacecraft must be tightly controlled during science observations. The spacecraft need to align the formation with cm-level position accuracy, with 30 arcseconds of angular precision, and a drift rate less than 0.25 arcseconds/second. [8] To achieve the desired relative position accuracy, corrective impulses must be performed throughout the orbit and within a minute prior to an observation. These maneuvers are intended to reduce the effects of unmodeled perturbations in the spacecraft trajectory, and as such, each maneuver is a stochastic impulse in direction and magnitude. [7] This impulse requires high accuracy and precision, but of concern in this analysis is the attitude of the spacecraft 30 seconds after the impulse. This attitude must settle to the required angular velocity and position within 30 seconds after the impulse maneuver is completed to meet all positioning requirements in time for a sequence of images. [8]

Achieving this angular accuracy within this timeframe is possible using commercially available components, as demonstrated by the Asteria mission. [9] VISORS will use a commercial spacecraft bus with similar capabilities. However, to perform the required proximity operations, the VISORS spacecraft are additionally equipped with thrusters. This adds more complexity to the system, as the thrusters used by VISORS store propellant as a liquid. [1] The problem of fuel slosh becomes relevant to achieve precise attitude control. As a commercial-off-the-shelf system, this kind of spacecraft bus is typically installed with only simple attitude controllers with limited customization. Thus, it is crucial for mission success to determine the vehicle's response to the actuation of the thruster.

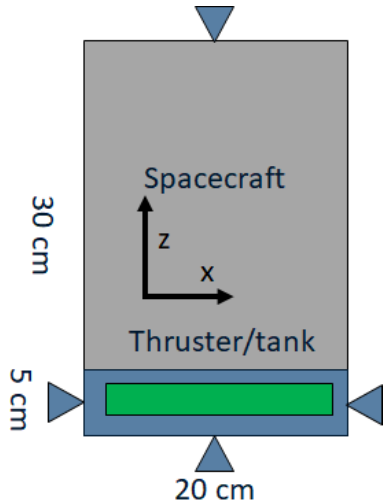

Figure 2: VISORS Spacecraft Propulsion System Model

The propulsion system used for VISORS is a cold-gas thruster which is modified from the thruster used by NASA's BioSentinel mission. [1] The propellant is R236fa, which is a common refrigerant. The propellant is stored as a pressurized liquid in a primary tank. Prior to maneuvers, the liquid is vented into a secondary tank to evaporate before being expelled to produce thrust. The location of the propulsion system is particularly important for analyzing the slosh response. As seen in Figure 2, the thruster is located on the extreme end of the spacecraft, approximately 15 cm from the geometric center of the spacecraft. With multiple nozzles pointed in different directions, this thruster has the capability to provide an impulse in any direction by actuating nozzles in couples. The propellant storage tank is approximated as a cylinder with equivalent length and volume.

In addition to the science mode, it is important for this mission to understand other operations which may be affected by propellant slosh. These operations have been identified as any event which imparts a consistent acceleration to the liquid or results in relatively high velocity fluid motion. For VISORS, these additional operations have been identified as: reorientations for large impulse maneuvers, slewing to track ground stations, plenum refilling, and large impulse maneuvers for orbit reconfiguration. These situations do not require nearly as precise pointing accuracy, but they are still analyzed, as the results of this paper aim to be generally applicable to a range of requirements.

The analysis completed on CubeSat propulsion systems is done using the parameters of the VISORS system. This thruster contains a propellant tank with a volume of 176 cc and uses R236fa as propellant. The relevant parameters of the thruster are given in Table I. The tank parameters are given from Stevenson's paper. [1] The dynamic viscosity and surface tension of R236fa were reported by Laesecke [10] and Mulero [11] respectively. Both properties decrease with an increase of temperature, so depending on the actual conditions of the spacecraft propellant, these values will vary significantly. The nominal values are chosen for an ambient temperature of 290K. R236fa has moderate surface tension and viscosity. Compared to water at 290 K, which has a dynamic viscosity of 1 mPa•s, and a surface tension of 70 mN/m, R236fa has lower viscosity and surface tension. However, even with these lower magnitudes, the viscous and surface tension effects are important for this propulsion system, given the small volume of the system.

Table I: Propulsion System Parameters

Parameter	Value
Main tank volume	176 cc
Actuation force	20 mN
Approximate spacecraft mass	12 kg
Propellant	R236fa
Density	1270 kg/m^3
Propellant mass	220 g
Ambient temperature	290 K
Dynamic viscosity 260-300 K	0.46-0.27 mPa*s
Surface tension 260-300 K	14.4-9.4 mN/m

II. Slosh Modeling for VISORS

Slosh Dynamics Regimes

To easily approximate the characteristics of the slosh dynamics, the relative importance of the forces acting on the system must be determined. The main effects which can influence the dynamics present in the fluid tank are body forces, inertia, gravity, viscous forces, and capillary effects. Body forces can result in multiple types of response. If the body is undergoing an external acceleration, such as a thruster impulse, this is modeled as an acceleration in the tank frame. Other responses to body forces can be characterized by the body slewing to change attitude. If one or more of these effects can be shown to dominate the system response, the other effects can be approximately ignored. [5] Typically, viscous forces are always important in the response, as there must be damping for the system to settle. The viscous effects can be added onto the overall response. The regimes which characterize the response can be summarized as inertially, gravity, or capillary dominated.

To determine which regime is dominant for a specific disturbance, three dimensionless parameters are used: the Bond, Froude, and Weber numbers. These parameters are defined in equations 1-3 [5]:

$$We = \frac{Inertia}{Capillary} = \frac{\rho V^2 L}{\sigma} \tag{1}$$

$$\text{Bo} = \frac{\text{Gravity}}{Capillary} = \frac{\rho a L^2}{\sigma} \tag{2}$$

$$\text{Fr} = \frac{\text{Inertia}}{Gravity} = \frac{V^2}{aL} \tag{3}$$

where ρ is the fluid density, L is a characteristic dimension of the tank (typically the radius), V is the relative fluid velocity, σ is the surface tension, and a is the relative acceleration of the fluid in the body frame, not inertial acceleration. [5] These three equations follow the relation: $We = Fr*Bo$. Thus, there are only two independent parameters. However, three parameters are used to provide a direct comparison between two regimes.

By computing these dimensionless parameters for different operational considerations, the dominant regime can be determined, if one exists, and if not, the comparison allows for determination of relevant model parameters. For this paper, a dominant regime is assumed to be a condition under which the magnitude of the forces from one effect are double the magnitude of both other effects. Once a dominant regime is determined, an approximate model can be derived. Furthermore, experimental results for fluid response are typically only valid for one regime. [5] Thus, determining the regime allows the use of experimental derivations and simpler models. Example calculations for these regimes determine factors such as the natural frequency and time constant of the response.

Another factor for experimentally derived results is the shape of the fluid tank. The shape of the container becomes particularly important when accounting for capillary effects, as it defines the possible fluid interfaces. A key aspect of the capillary-dominated regime is this fluid interface and the energy associated with it. This energy is called the capillary potential energy, which is defined in equation 4. [5]

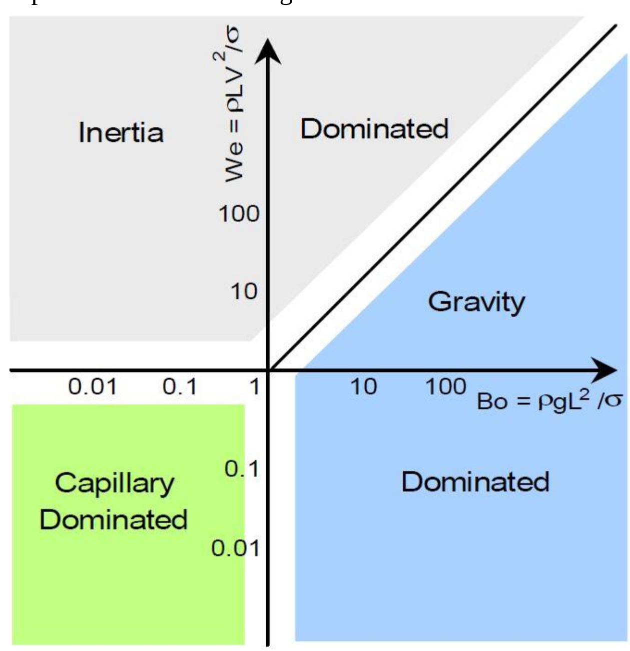

Figure 3: Force regimes using dimensionless parameters [5]

$$PE_c = \sigma A_C = \sigma(A_i - \cos\theta_c A_{wet}) \tag{4}$$

where σ is the surface tension, A_i is the vapor-liquid interface area, A_{wet} is the solid-liquid wetted surface area, and θ_c is the contact angle, a measure of how well the surface absorbs or wets the surface. At the minimum potential energy, the system is in a stable equilibrium. This relation gives significant insight into the shape of the surface, as the capillary-dominated system will be at a stable equilibrium when the vapor-liquid interface area is minimized. This means that the propellant's minimum energy state is in a single liquid drop with minimal air contact. An additional property of capillary forces is that the capillary forces will work to bring the system to a local minimum potential energy, which means that there exist multiple metastable propellant shapes and at least one globally stable propellant configuration. This potential energy equation is especially relevant when combined with gravitational potential energy in equation 5, which can be used to define the effect of an impulse on the stable liquid shape: [5]

$$PE_{tot} = PE_g + PE_c = mah + \sigma(A_i - \cos\theta_c A_{wet}) \tag{5}$$

where *m* is the fluid mass, *h* is the height above a reference surface, and *a* is the relative acceleration of the fluid. It is important to note that this equation is valid only during constant relative acceleration. If the direction of the inertial relative acceleration is changing, this equation is no longer valid. Thus, equation 5 is only used instantaneously or with constant direction acceleration.

Slosh Dynamics Response for the VISORS Configuration

Applying the slosh modeling of the previous section to the VISORS mission provides a method for understanding the dominant fluid regime for different operational aspects of the mission. The VISORS propellant tank can be modeled as a cylinder spanning the width of the spacecraft with a height of 15 cm and a radius of 2 cm. Under microgravity conditions, a number of semi-stable interfaces are possible. For a contact angle of 90 degrees, some example interfaces can be seen in Figure 4. The contact angle that R236fa makes with the surface is unknown, so these interfaces and their stability could be very different.

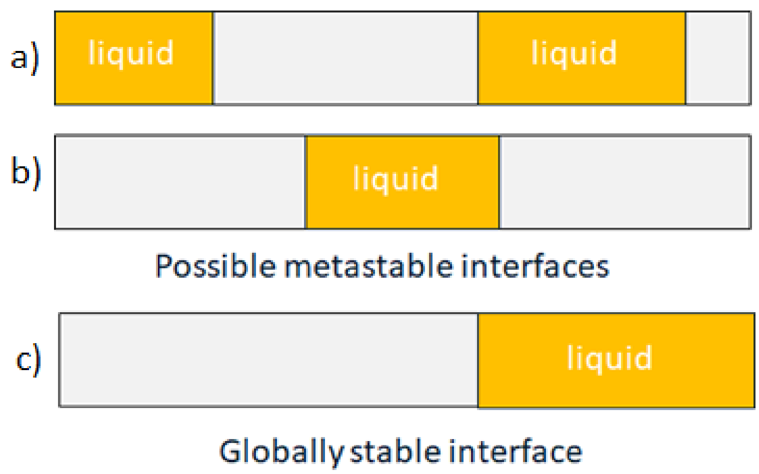

Figure 4: Possible Interface Conditions for a Cylindrical Propellant Tank a) 1 stable, 1 metastable interface b) 1 metastable interface c) 1 globally stable interface

Immediately it can be seen that the globally stable condition for a cylindrical tank would primarily disturb the z-axis moment of inertia, with the magnitude dependent on the fill level of the tank. It is hypothesized that a configuration with two unevenly distributed liquid bubbles on either side of the cylinder is the most likely metastable condition (shown in Figure 4a), as there is a significant increase in interface energy to detach the liquid from the tank walls. With the bubble in the middle, its location is independent of its potential energy, so small disturbances would likely perturb it until it goes to the side of the tank.

In nominal conditions, the primary relative acceleration acting on the spacecraft-fluid system is a combination of drag, gravity gradient, and other perturbations. The magnitude of this acceleration is estimated to be approximately 10^{-5} m/s^2. Using equation 2 provides a Bond number of 4.4×10^{-4}. Thus, under nominal conditions, the surface tension has a much more significant effect than external accelerations. Under nominal conditions, the relative velocity of the fluid is zero, so there is no Froude or Weber number. Thus, the nominal regime is the capillary-dominated regime for the VISORS spacecraft, and the nominal fuel configuration is in a metastable condition. Note, for alternate tank shapes, such as a sphere or ellipsoid, every interface may be metastable, as the interface area could be the same regardless of the rotation about the sphere. Thus, there would be no globally stable interface and capillary effects would likely not have as significant a role.

When the VISORS spacecraft performs a slew maneuver or an impulsive maneuver, these equilibrium conditions are disturbed. For a rapid slew maneuver of 10 deg/s, the fluid may possibly go through all three regimes. When slewing at 10 deg/s, an acceleration of 4.7×10^{-9} m/s^2 is seen in the tank frame according to the vector transport theorem, using equation 6.

$$\frac{{}^I d^2}{dt^2}\bar{r}_{ab} = {}^B\ddot{\bar{r}}_{ab} + 2\,\bar{\omega}^{B/I}\, x\, {}^B\,\dot{\bar{r}}_{ab} + \dot{\bar{\omega}}^{B/I} x\, \bar{r}_{ab} + \bar{\omega}^{B/I} x(\,\bar{\omega}^{B/I} x\bar{r}_{ab}) \tag{6}$$

The acceleration of the slosh, after the rotation has reached equilibrium, can be given by assuming $\dot{r} = \ddot{r} = \alpha = 0$ in equation 6, with $\bar{r}_{ab}$ as the vector between the tank and the center of mass. This gives equation 7

$$\frac{{}^I d^2}{dt^2}\bar{r}_{ab} = \bar{\omega}^{B/I} x(\,\bar{\omega}^{B/I} x\bar{r}_{ab}) \tag{7}$$

where $\bar{\omega}^{B/I}$ is the angular velocity of body frame with respect to inertia, $\bar{r}_{ab}$ is the distance between the spacecraft center of mass and the tank geometric center, *I* refers to the inertial frame, and *B* refers to the spacecraft body frame.

The resulting Bond number for this condition is 0.2, showing that there is a small disturbance to the capillary equilibrium conditio1n. An increase in the tank's characteristic length (the radius) from 2 to 4 cm (4 times the volume), would be required for a Bond number of 1 and equal relevance of acceleration forces and capillary forces.

When the slew maneuver is started or stopped, the torque of the spacecraft is limited by the reaction wheels, so the relative acceleration of the liquid is limited. This can be computed with equation 8 by simplifying equation 6 with the assumptions ${}^B\ddot{\bar{r}}_{ab} = {}^B\,\dot{\bar{r}}_{ab} = \bar{\omega}^{B/I} = 0$.

$$\frac{{}^I d^2}{dt^2}\bar{r}_{ab} = \dot{\bar{\omega}}^{B/I} x\bar{r} \tag{8}$$

For the minor inertia and max torque of the VISORS spacecraft, this acceleration has a magnitude of 1.1e-2 m/s^2. The Bond number is 0.47 which means the capillary and rotational accelerations have similar values. This result suggests that the system is sensitive to rotational disturbances, and if the spacecraft uses its maximum torque capabilities, the capillary equilibrium may be disturbed. However, even in a worst-case scenario where the tank size is much smaller, this acceleration does not dominate the system, with Bond numbers not exceeding 2.

When the pressure of the gas in the plenum falls below a desired value, approximately 80% of the original pressure, fluid is transferred between the propellant tank and the plenum. This is the only disturbance to the fluid which does not occur due to body forces. Thus, depending on the magnitude of the plenum refill, the inertial properties may be dominant. For the VISORS spacecraft, the mass flow rate into the plenum is approximately 30 mg/s. This value is much smaller than the total mass of propellant, at 200,000 mg, so it is not expected to be a significant source of error.

When the spacecraft undergoes an impulsive maneuver, the vehicle's relative acceleration is simply the magnitude of the thruster's acceleration. The thruster does not impart a significant relative velocity unless the fluid is floating freely for some period of time in the direction of the acceleration. Thus, the main influence an impulse, sustained or brief, has on the system is the acceleration which it provides to the system. With a nominal thrust force of 20 mN and a mass of 12 kg, the thruster only produces an acceleration of 1.7e-4 *g*. This corresponds to a Bond number of 0.08, which suggests that this

Table II: Bond number for specific spacecraft maneuvers

Condition	Bond # (cylindrical)	Bond # (spherical)
Nominal (μgravity)	4.4E-4	1.4E-3
Rapid Slew (10 deg/s)	0.205	0.66
Full Torque (7 mNm)	0.47	1.52
Short Impulse (20 mN)	0.0748	0.24
Impulse Torque (3 mNm)	0.201	0.65

acceleration is almost negligible for this system. However, when providing a torque to this system, the thruster produces a greater relative acceleration, due to the relatively low moment of inertia.

These results are summarized in Table II. The analysis also includes the case where the tank is modeled as a sphere with equivalent volume, providing an upper bound on the characteristic length. In conclusion, the primary regime of this system is the capillary regime, although relative acceleration caused by rotational motion is also a concern.

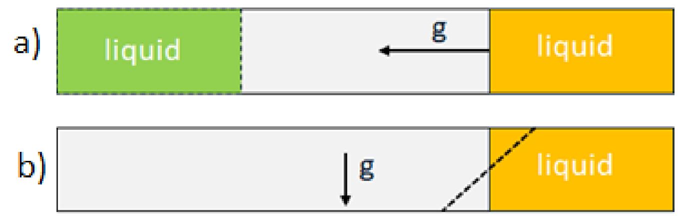

Figure 5: Minimum Energy Slosh Configurations Under Acceleration a) Slosh Inversion b) Disturbed Equilibrium

Because the capillary regime is largely dominant, these operations can be thought of as disturbances to the equilibrium interface. Depending on the direction of the disturbance in the tank system, the response will vary significantly. For example, as seen in Figure 5, the globally stable equilibrium conditions for the interface are dramatically different based on the direction of the acceleration. Once the disturbance acceleration ceases, the energy difference between the new interface and the original equilibrium may be entirely converted into vibrational energy. This energy must then be dissipated by damping before the system will remain stable. The worst-case scenario for this disturbance is the condition of interface inversion, which is where the acceleration is large enough to break surface tension in a direction which would require the fluid to fall, as seen in Figure 5a. Dodge has shown that the interface for a cylinder can invert when the Bond number is approximately 0.8 for a contact angle of 0 degrees and approximately 3.5 for a contact angle of 90 degrees. [5] Since the contact angle of the VISORS propulsion system is unknown, it is possible for this condition to occur under the example conditions. A simple way to estimate the amount of time it could take for this inversion to occur is to determine the time for the fluid's center of mass to free fall at the input acceleration to its new center of mass location, then multiply this value by 4 to allow time for the fluid velocity to be damped by viscous effects. [5]

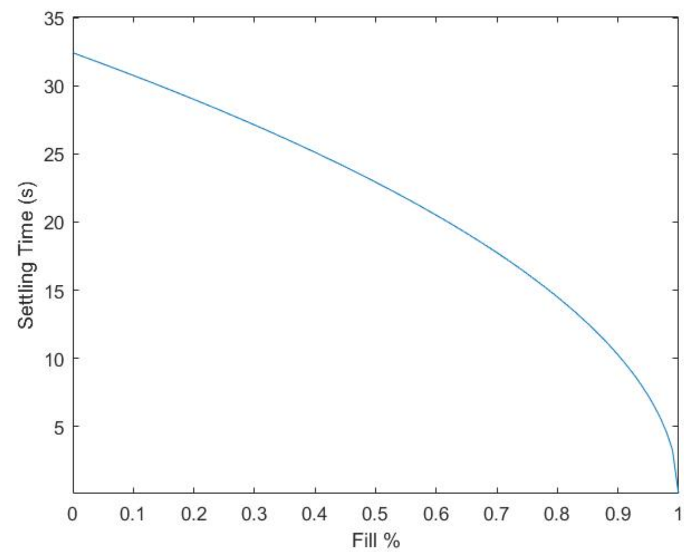

Figure 6: Settling Time Estimates for Fluid Inversion Under Constant Thrust

For an acceleration acting along the tank's x-axis, the settling time for an inverted interface is computed as a function of fill level. This analysis can provide a worst-case estimate for the settling time, assuming the acceleration is sufficiently large to destabilize the interface. For the case where the VISORS maximum thrust is given in the x direction, these worst-case settling times are given in Figure 6.

If the acceleration is stopped before the fluid has reached its new position, the fluid could retain a significant velocity, and the dynamics would be dominated by inertial, rather than capillary effects. For the VISORS mission, this is likely to occur when the VISORS control system exerts a constant torque for only a few seconds and stops the torque before reaching the inverted state. It can be seen in Figure 7 that, for an impulsive acceleration, the inertial response after that acceleration quickly dominates the response of the system. It is important to note, though, that this is only valid if this acceleration is sufficiently large (the Bond number is at least 2) to ignore the capillary force restraining the response. For

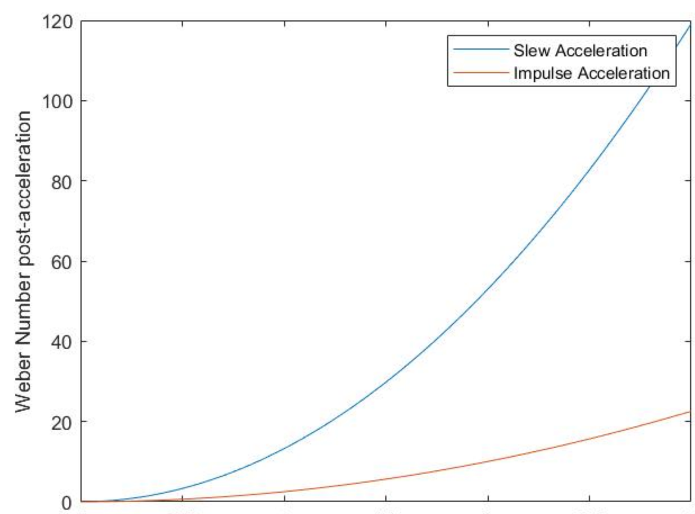

Figure 7: Worst-case Weber Number After Maneuver Acceleration

VISORS, this is not the case, and the capillary forces would significantly reduce the resultant Weber numbers. This condition is then useful for a worst-case scenario. This means that it is necessary to analyze the inertial response of this system in addition to the purely capillary response to bound the possible worst-case behavior of the system.

To estimate the amount of time that it could take to reform the capillary interface after transitioning out of a high acceleration environment, experimental results have been correlated to a model for cylindrical tanks, equations 9 and 10, where v represents the liquid kinematic viscosity (dynamic viscosity/density) and θc is the contact angle. [5]

$$t_s = \frac{R_o^2}{v} * \frac{10^B * \zeta^A + 0.01\alpha^2}{1+\alpha^2} \tag{9}$$

$$\alpha = \frac{1-\sin(\theta_c)}{\cos(\theta_c)} \quad A = 0.28 + 2.2\alpha - 1.2\alpha^2 \quad B = 3.9A - 3.32$$
$$\zeta = v * \left(\frac{\rho}{R_o \sigma \alpha^2 \cos(\theta_c)}\right)^{\frac{1}{2}} \tag{10}$$

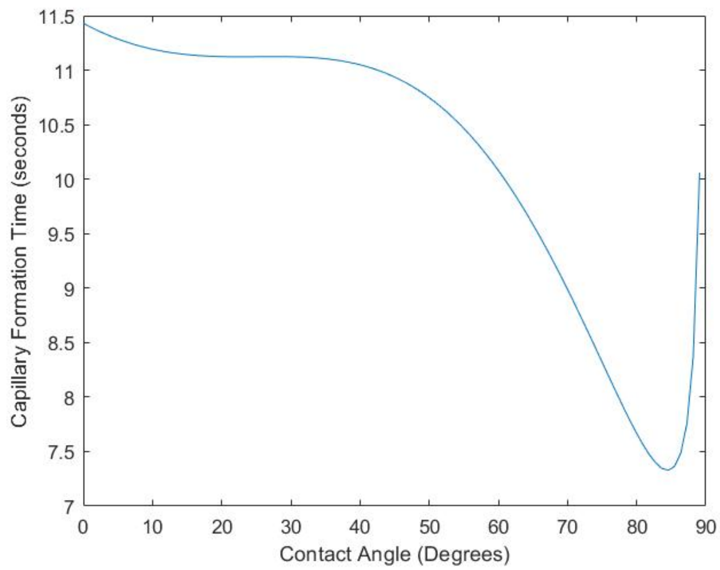

Figure 8: Theoretical Capillary Formation Time Post Disturbance

For the VISORS spacecraft, the capillary formation time was found for contact angles from 0 to 90 degrees. The capillary formation time is the amount of time that the fluid takes to reach a metastable condition after a change in the frame's relative acceleration. This provides a first order estimate for the settling time of the slosh after a thruster impulse. Figure 8 shows that, for higher contact angles, the formation time is about 60% of the low contact angle times. This result predicts that the propellant motion will damp significantly after about 10 seconds and that high contact angle propellant will settle significantly sooner than the low contact angle.

In summary, there are a range of possible responses depending on the actual contact angle of the system. For a contact angle of 0 degrees, disturbances which do not exceed a Bond number of 0.8 will likely have a settling response on the order of 12 seconds. However, if the Bond number exceeds this value, it could take as long as 35 seconds for the fluid to settle, as shown in Figure 7. In contrast, if the system has a contact angle of 90 degrees, it requires a disturbance with a bond number of 3.5 to significantly disturb the system's equilibrium, which allows accelerations ~5 times higher to be ignored as significant disturbances. Thus, a crucial unknown in this analysis is the contact angle the R236fa fluid makes with the tank wall material. This has a substantial effect on the result of the analysis. In addition, with the quick capillary formation time shown in Figure 8, it is a reasonable assumption that capillary forces need to be modeled throughout this analysis.

Analogous Mechanical Model

The previous slosh analysis is useful to grasp an understanding of the importance of the different forces involved, and it can provide estimates of the response time of the system. However, that analysis does not give a time response of the system and cannot be easily analyzed for conditions with additional dynamics to consider. Thus, to apply the fuel slosh analysis to the attitude control of CubeSats, a physical model is needed. Analogous mechanical models commonly consist of either a spring-mass-damper system or a mass pendulum system. [5] The parameters are chosen by using experimental models to determine system properties such as the natural frequency of the motion and the mass fraction. Once an equivalent mechanical model has been constructed within the tank, the dynamics of the system and how it interacts with the rest of the satellite can be evaluated. This model is very similar to De Souza's, [6] as it only considers 2D motion and uses a similar configuration. However, the previous system presented assumes that the motion is dominated by gravity forces by using a constantly actuating thruster. Since this paper has determined that CubeSat propulsion system typically operates in the capillary regime, the model with constant external acceleration will not have the same dynamics.

The slosh dynamics are modeled using a spring-mass-damper pendulum, with the fulcrum of the pendulum rigidly connected to the spacecraft. The spring is meant to model capillary effects, the damper models the fluid's viscosity, and the pendulum mass is located at the propellant center of mass. In this model, there are three frames: the slosh mass frame, the spacecraft frame, and the inertial frame. The relation between the frames is given in equation 11. It is important to define the slosh frame as being relative to the body frame, as the relative motion between the body and slosh frame defines the dynamics of the motion.

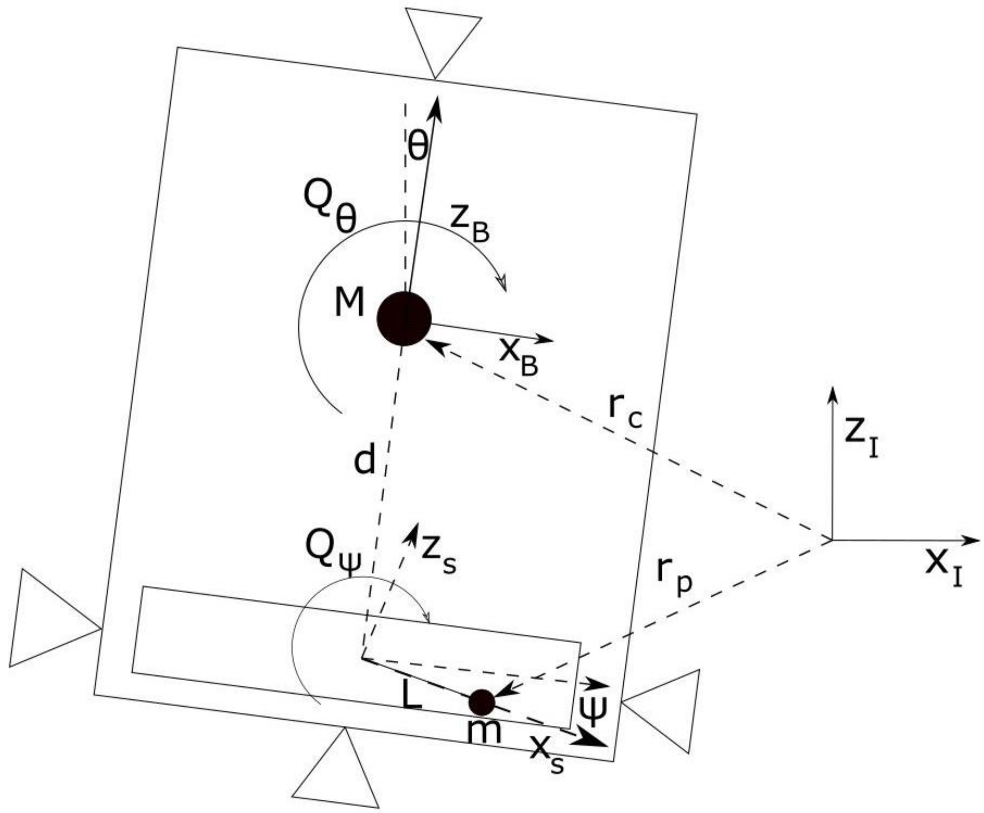

Figure 9: Spacecraft and Fluid Tank Model

$$\overline{\omega}^{B/I} = \dot{\theta}\hat{y}, \quad \overline{\omega}^{S/B} = \dot{\psi}\hat{y}, \quad \overline{\omega}^{S/I} = (\dot{\theta} + \dot{\psi})\hat{y} \tag{11}$$

This model, shown in Figure 9, is analyzed using Lagrangian dynamics. The generalized coordinates of this model are θ, ψ, x, and z. The velocities of this system are given by equations 12 and 13.

$$\dot{r}_c = \dot{x}\,\hat{x}_I + \dot{z}\,\hat{z}_I \tag{12}$$

$$\dot{r}_p = \dot{r}_c + \omega^{B/I}\, x\, (-d\hat{z}_B) + \omega^{S/I}\, x\, (L\hat{x}_s) = \dot{r}_c - \dot{\theta} d\hat{\imath}_B - (\dot{\theta} + \dot{\psi})L\hat{z}_s \tag{13}$$

The resulting Lagrangian, with the added angular velocity of the rigid body spacecraft, is given in equation 14. The generalized forces are given in equations 15-18.

$$L = \frac{1}{2}(M+m)(\dot{x}^2+\dot{z}^2) + \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}m(\dot{\theta}^2 d^2 + l^2(\dot{\theta}^2+\dot{\psi}^2) + \frac{1}{2}m(-\dot{\theta}\dot{x}dcos(\theta) + \dot{\theta}\dot{z}dsin(\theta) - \dot{x}(\dot{\theta}+\dot{\psi})lsin(\theta+\psi) - \dot{z}(\dot{\theta}+\dot{\psi})lcos(\theta+\psi) + ld\dot{\theta}(\dot{\theta}+\dot{\psi})sin\,(\psi)) \tag{14}$$

$$Q_x = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} \tag{15}$$

$$Q_z = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} \tag{16}$$

$$Q_\theta = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} \tag{17}$$

$$Q_\psi = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\psi}}\right) - \frac{\partial L}{\partial \psi} \tag{18}$$

Where Q_i are the generalized forces, *M* is the spacecraft mass, *m* is the slosh mass, *d* is the distance from the spacecraft geometric center to the fulcrum of the slosh pendulum, and *l* is the length of the pendulum to which the slosh mass is attached.

This model provides a system of equations in terms of the generalized forces which are linear in the second derivatives of the generalized coordinates. The generalized forces are used to model external forces and the slosh pendulum response. The dynamics do not use a small angle approximation, though this system will not operate at large angles, as it is desired to have as accurate a system as possible to predict the response with high precision. The fuel slosh tank is modeled as a cylinder, using the same parameters presented earlier. The model is not valid for large angles of ψ, as the movement of the liquid would be restrained by the walls of the tank.

The generalized forces are expressions of external torques and forces. Q_x and Q_z both result from the thruster exerting a force in the body frame. Q_θ is the torque exerted by the attitude control system on the spacecraft. With Q_ψ the external source is the fluid itself with the energy damped by the fluid motion. The other torques are dependent on the specific configuration of the problem, specifically the actuators and control laws being used. For the VISORS mission, the spacecraft is assumed to be using a proportional-derivative (PD) control law to control the body rotation angle θ with reaction wheels, and it is using the thrusters to control the *x* and *z* position. These values result in the generalized forces given in equations 19-21.

$$Q_x = F_{x_B}\cos(\theta) + F_{z_B}\sin(\theta) \tag{19}$$

$$Q_z = -F_{x_B}\sin(\theta) + F_{z_B}\cos\,(\theta) \tag{20}$$

$$Q_\theta = -k_d\dot{\theta} - k_p\theta - F_{x_B}d \tag{21}$$

Q_ψ depends on the specific parameters of the problem which are shown in following sections. Since the thrust forces are defined in the spacecraft frame, the force requires a rotation to map it to the inertial frame. Besides the control torques acting on θ, the propulsive force along the body *x*-axis generates a torque about θ.

With the mechanical model defined, the parameters need to be determined. This is where the previously shown, experimentally based fluid representations become very useful. They allow for validation of the mechanical model and for providing relations between the tank and fluid properties and the constants of the system, specifically the length, spring constant, and damping ratios.

Typically, two masses are associated with the slosh motion. One is a stationary mass and the other is a moving mass. Depending on the shape and fill level of the container, the percent of the mass that can move can be as low as 5-10% of the total slosh mass. [5] However, to simplify the parameter derivation while still providing a worst-case analysis, the entire slosh mass is considered to be mobile. Furthermore, the location of the slosh mass has a significant effect on how the system interacts with the spacecraft dynamics, so the length of the pendulum is chosen to be the distance from the pendulum hinge point to the center of mass of the propellant. To simplify the model further, the propellant mass is treated as a single large mass in the minimal interface condition. Thus, the pendulum's equilibrium position is simply given by equation 22 and the mass is given by equation 23

$$l = \frac{1-h}{2} \tag{22}$$

$$m = \frac{h}{h_{cyl}} * m_{max} \tag{23}$$

where h_{cyl} is the maximum cylinder height, h is the propellant height, and m_{max} is the thruster's maximum filled propellant mass.

Choosing the length and mass based on the fill percent is useful because the length of the slosh mass's moment arm directly correlates with the impact the slosh mass has on the rest of the body. For high fill percentages, the tank is full and the fluid does not have much space to move.

To determine the spring constant and natural frequency of the system, it is important to know the natural frequency of the slosh motion. In this case, only the first natural frequency of the motion is of concern, as this mode contains the majority of the liquid's mass and has the longest period of the liquid motion. For a cylindrical tank, assuming a contact angle of 0 degrees, the expression for the first natural frequency of the motion is approximated by equations 24 and 25 [5]

$$h > 3r: \quad \omega_1 = 1.61 \left(\frac{\sigma}{\rho r^2}(1 + 0.798Bo\right)^{\frac{1}{2}} \tag{24}$$

$$h < 3r: \quad \omega_1 = 1.61 \left(\frac{\sigma}{\rho r^2}(1 + 0.798Bo)\right)^{\frac{1}{2}} * \tanh\left(\frac{1.841h}{r}\right) \tag{25}$$

where *r* is the radius of the cylinder and *Bo* is the Bond number associated with the fluid motion. For a contact angle of 90 degrees, this relation is computed by equation 26 where *g* is the relative acceleration of the fluid in its tank.

$$\omega_1 = sqrt\left(\left(\frac{3.3893\sigma}{\rho r^3} + \frac{g}{r}\right) 1.841 \tanh \frac{1.841h}{r}\right) \tag{26}$$

For an example fill level of 0.4, the natural frequency for the 0 degree contact angle is ω_1 = 0.23 rad/s whereas the 90 degree contact angle has a natural frequency of ω_1 = 2.54 rad/s, an approximately 10 times increase. This is a very significant variation as this frequency also effects the damping ratio according to equation 27 (Dodge)

$$0 \leq Bo \leq 1: \quad \gamma = 4.47\sqrt{\frac{\upsilon}{\omega_1 r^2}} \tag{27}$$

These relations depend on the relative acceleration of the slosh. For the analysis where there are no external thrusts and only small moments acting on the system, the acceleration is found to be on the order of 10^{-6}, so it is ignored in this analysis. However, for a more accurate dynamics model, it would be necessary to create a state-dependent spring constant and natural frequency.

From linear systems theory, a pendulum with a torsional spring and torsional damping can be described in equation 28. The spring constant and damping coefficient are then found to force the slosh system to follow the desired response.

$$\frac{d^2\psi}{dt^2} + 2\gamma\omega_n\frac{d\psi}{dt} + \omega_n^2\psi = 0 \tag{28}$$

$$c = 2\gamma\omega_n m l^2 \quad k = \omega_n^2 m l^2 \tag{29}$$

$$Q_\psi = -c\dot{\psi} - k\psi \tag{30}$$

The generalized torque Q_ψ, comes from the spring mass damper model. This results in a moment about the slosh pendulum which is added with the other generalized forces to the equations of motion.

Important assumptions of this model are: (1) small angle slosh perturbation, (2) the relative acceleration does not exceed a Bond number of 0.8, which provides a lower bound on the acceleration which could destabilize the equilibrium condition. The remaining parameters of this system are the temperature, fill level, and contact angle.

III. VISORS spacecraft attitude analysis

For the VISORS mission, the primary concern for attitude settling time is that, just after a, the spacecraft will take a disproportionately large amount of time to settle due to the motion of the slosh disturbing the spacecraft. To study this case, a simulation of the attitude control system was set up with the previously derived dynamics, and control gains were selected for the spacecraft PD controller to provide settling times of approximately 15 seconds. The control torque was limited to lie within the system capabilities. The parameters used for this analysis are listed in Table III. Since the damping and spring coefficients vary with mass, example coefficients are given. Typically, only a portion of the propellant is free to move. However, it is not well understood, so for these analyses, it is assumed that the entire propellant slosh mass can move, which means this analysis is a worst-case scenario.

Table III: Parameters for Mechanical Model

Parameter	Value
Spacecraft mass	12 kg
Spacecraft moment of inertia	0.13 kgm^2
Maximum propellant mass	220 g
Distance between pendulum and SC CoM (d)	15 cm
Contact Angles	0° (perfectly wetting) and 90° (neutral wetting)
Maximum length of pendulum	0.15 m
Tank Dimensions (cylindrical)	1.9 cm radius, 15 cm height
Controller gains	kp = 0.2, kd = 0.2
Example damping coefficient (90°,0°)	6.2e-5 , 1.8e-5
Example spring constant (90°,0°)	1.3e-3 , 1.0e-5

To simulate the spacecraft formation adjustment maneuvers, the spacecraft was commanded to accelerate in the body *x* direction for 6 seconds while the attitude control system was active. This gives an impulse of 1 cm/s to the spacecraft. This is much larger than the expected corrective maneuvers, so this response will bound the perturbation. The propulsion tank's fill level was set to be 40%. The system was simulated under two different conditions: (1) a contact angle of 0 degrees, the fully wetted condition, and (2) a contact angle of 90 degrees, the neutrally wetted condition. The response in the body angle, θ, and the slosh angle, ψ are shown for these cases in figures 10 and 11. In each case, the response of θ and $\dot{\theta}$ appear to be nearly the same, while the slosh angle, ψ, has a very different response. The response of ψ is different for each of these conditions because the contact angles have frequencies an order of magnitude apart. This is the most striking difference between the two simulations, but on a macroscopic level, they do not appear to have a noticeable impact on the spacecraft's body angle, θ.

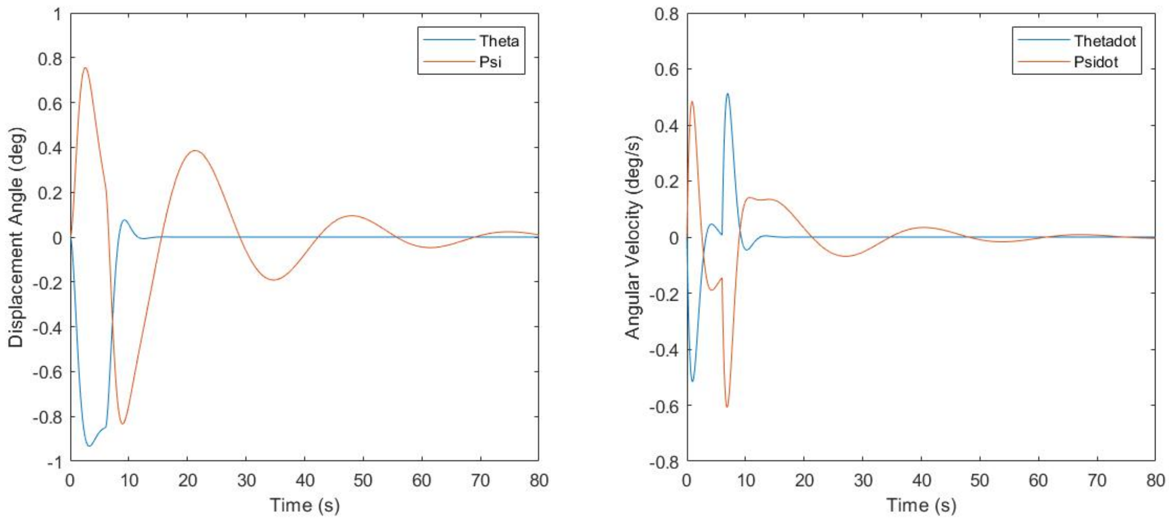

Figure 11: System response to 6 second impulse when the tank has a 0 degree contact angle

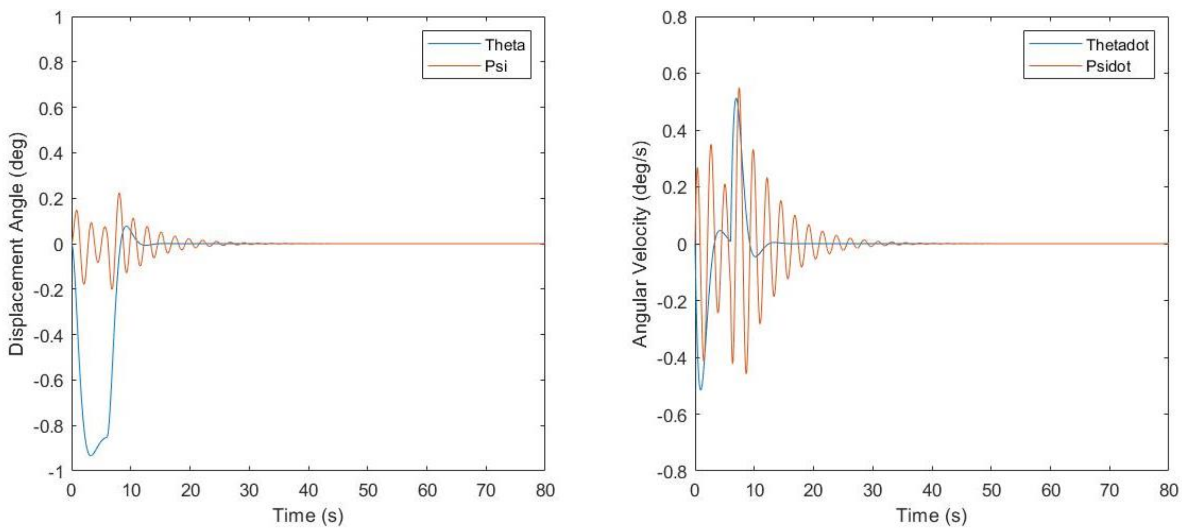

Figure 10: System Response to 6 second impulse when the tank has a 90 degree contact angle

However, at the accuracies required for the VISORS mission, the slosh motion is enough to make a significant difference. In figure 12, the 90 degree contact angle slosh is compared with the 0 degree slosh, comparing the angular velocity of the spacecraft body angle, θ, at the relevant timescale. This shows that there are very different responses in the velocity of these two systems, resulting in significantly different settling times between the two sets of dynamics.

The settling time of the system is defined as the time it takes to reach the required angular position and velocity after a disturbance. The angular velocity of the system takes much longer to damp out, so it is used as the settling time of the system. Figure 13 shows how this settling time varies with fill level for three cases with the designed maximum propellant mass: slosh with a contact angle of 0 degrees, slosh with a contact angle of 90 degrees, and no slosh motion. This response shows that the 90 degree motion has a longer settling time than the 0 degree motion. This variation in settling times is significant, but it would not significantly impact the mission. The jumps in the settling times are approximately a single period long, resulting from the settling time. Figure 14 represents the response of this system if the mass of the propellant tank is doubled, all other parameters kept the same. This response is significantly worse than the original result but only for the 90 degree contact angle system. This suggests not only that the system is highly sensitive to the propellant mass but that the system is very sensitive to the contact angle as well.

This response is the culmination of many other parameters and varying them slightly causes a significant effect on the system. It is also important to note that these settling times are dependent on the gains of the spacecraft control system. For lower gain systems with longer settling times, the 0 degree contact angle case has larger settling times than the 90 degree case, as the latter's higher frequency results in damping occurring much sooner. The lower damping and frequency for the 0 degree case means that the system oscillates much longer and can still cause significant oscillations if the controller gains are not sufficient to respond to the system.

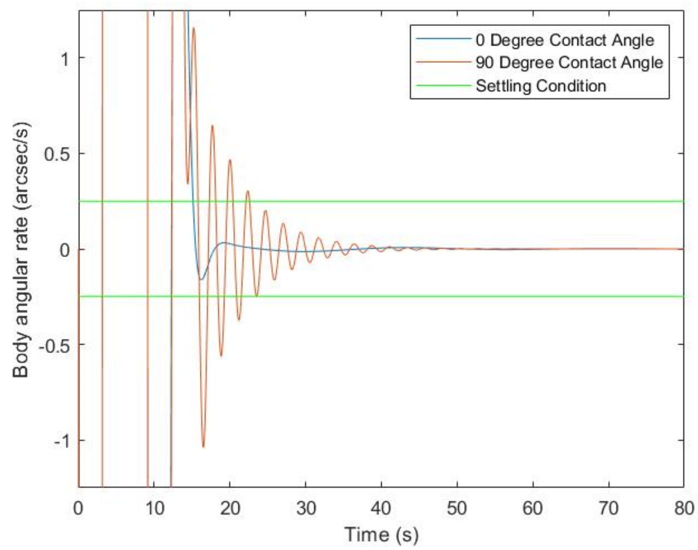

Figure 12: Comparison of precise pointing accuracy between contact angles

From the slosh regime analysis in Section II, the impulse event was shown to have a moderate disturbance on the system with regards to Bond number and the relative acceleration of the slosh mass. Performing an angular rotation had a slightly larger disturbance. Thus, it is important to examine the response of the system to angular rotations and see whether this disturbance has a greater impact on the settling times. To test this case, an angular rotation of 15 degrees was commanded using the same control gains as before. Once again, the response varied significantly with the contact angle. Figure 15 shows how the settling time of the system varied with the fill percent for these cases.

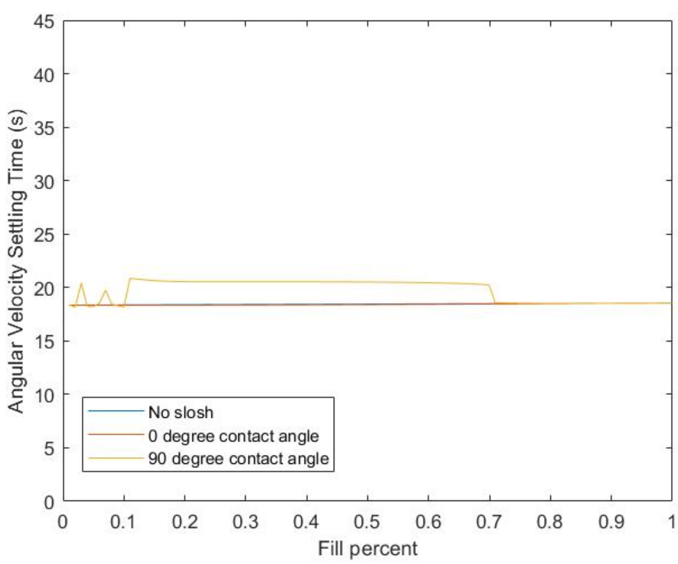

Figure 14: Settling time as a function of tank fill percent for design case and 220 g of propellant

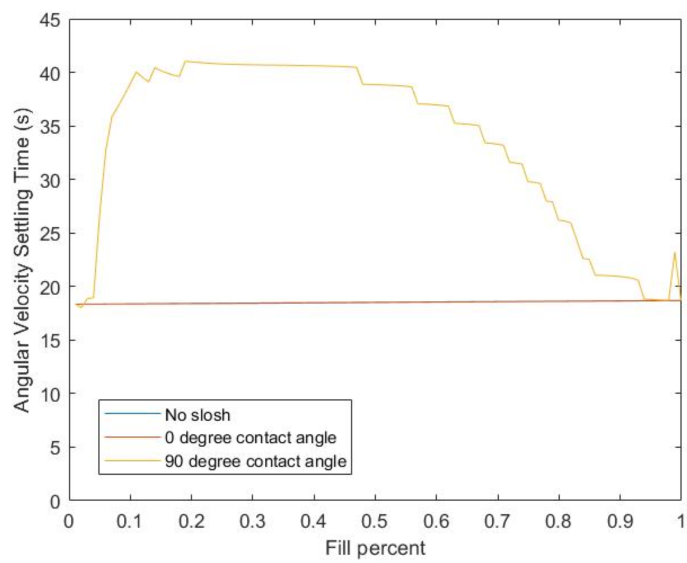

Figure 13: Settling time as a function of tank fill percent for 400 g of propellant

For this 15 degree rotation, it is evident that the slosh significantly perturbs the motion of the system, as slosh is typically causing 3 to 6 seconds of additional settling time. The effect becomes especially pronounced for the 0 degree contact angle case once the fill percent surpasses 80%. At this point, the settling time, only for the 0 degree contact angle case, increases rapidly with fill percent. It is hypothesized that this increase is related to the decreased moment of inertia of the model. However, it is not immediately obvious whether this matches real-world effects.

With a much larger slosh effect on the settling time of the rotating system, which is an ~25% increase at most fill levels compared to a 5% or 0% increase for the impulsive maneuver, the rotational motion of the system will be affected by slosh and it should be accounted for in operations and mission design. If at all possible, slew maneuvers should not be performed prior to high accuracy targeting.

An additional concern for the spacecraft is that the liquid distribution is skewed to a single side in its stable interface condition. This will unbalance the center of mass, which will reduce the accuracy of the impulsive maneuvers. This is a concern, though it is likely not a considerable concern, as the slosh mass is likely to be split into two fluid bubbles in the real case, which would reduce the center of mass imbalance. Even though the distribution of the fluid is random, if there are two fluid masses on either end of the cylinder, this condition will be more symmetric, causing smaller disturbances on the system.

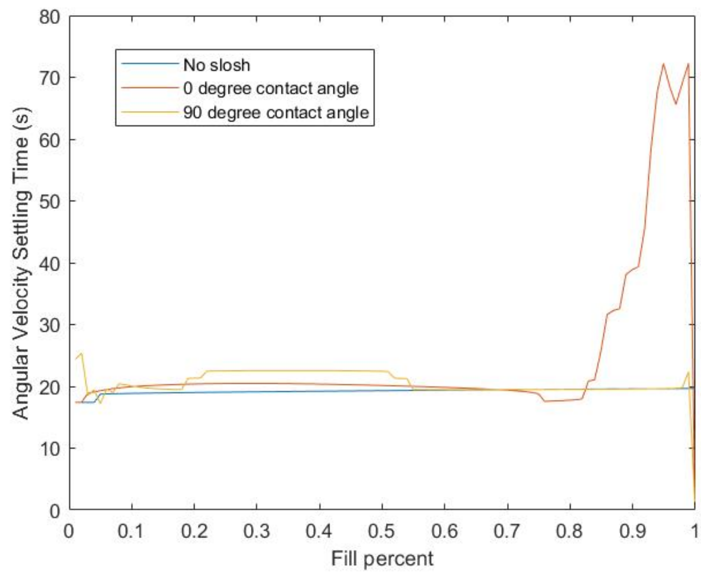

Figure 15: Settling times for 15 degree body rotation across fill percentages and contact angles

IV. Conclusion and Future Work

By starting from the dimensionless parameters of fluid motion, namely the Bond, Froude, and Weber numbers, a model was developed which is applied to the VISORS mission concept. From the relative magnitude of the forces acting on the system, it was found that, even while the spacecraft is actuating, the motion of the propellant is largely determined by the effect of capillary forces. This is largely due to the small tank container size, with a characteristic length of only 2 cm, as well as the small thrust magnitude of cold-gas propulsion. Thus, the pendulum model uses a torsional spring to model the fluid's tendency to return to a minimum stable interface. Using experimentally derived equations, the natural frequency of the system was found to have an order of magnitude difference depending on the wettability of the propellant on the tank wall.

The VISORS configuration was analyzed subject to these modified dynamics, and it was found that, for pointing applications with requirements greater than 5 degrees, the effect of the limited fuel slosh was negligible. However, for missions with precise pointing requirements, these disturbances can have a noticeable effect on the attitude control of the spacecraft. For the VISORS mission specifically, the spacecraft attitude response is largely undisturbed by impulsive maneuvers, given the small propellant volume and the small capability of the thruster. For a 6 second impulsive maneuver, the spacecraft took only 10% longer under worst-case conditions, and when accounting for the fact that only 10-30% of the propellant moves during slosh motion, the system should experience hardly any increase in settling time when an impulse is applied. Instead, the most important disturbance of operational concern is the response to any rotational maneuvers, with a 15 degree maneuver resulting in a 25% increase in settling time for most fill levels.

However, this analysis was performed using many assumptions on the system, particularly the control gains of the spacecraft. With different control gains, different results emerge due to the interaction between the slosh dynamics and the controller. This is largely dependent on the natural frequency of both the slosh response and the attitude control system and comparing these response frequencies would help identify control gains which could cause resonance and destabilize the system. Further, this analysis is very sensitive to changes in the mass, volume, and location of the propellant tank. Understanding the design constraints which fuel slosh imposes on different spacecraft missions would require a multivariable search to find the sensitivity of the system's settling time with respect to each variable. The main parameters to vary would be the size of the liquid fuel tank, the shape of the fuel tank, the temperature, the mass fraction, the fill level, spacecraft controller gains, and the contact angle.

To provide a more general analysis which could be suited to arbitrary tank shapes and dimensions, the potential energy of the minimum stability interface could be computed in the presence of gravitational perturbations. This energy, once compared with the unperturbed interface potential energy, would provide an upper bound on the amount of energy which must be damped by the fluid to re-stabilize the system after a disturbance. This would provide a general upper bound for the time it takes for the fluid to damp the spacecraft motion, without relying on mechanical modeling.

Since there was found to be a significant difference between the slosh response with different surface wetting conditions, the controller could be tested at a range of different conditions ranging from the 0 degree to the 90 degree response. If possible, the propellant contact angle should be determined experimentally to reduce the number of variables in the system.

V. Funding Sources

This research was funded by NSF grant 1936576.

VI. References

[1] Lightsey, E. G., Stevenson, T., and Sorgenfrei, M., "Development and Testing of a 3-D-Printed Cold Gas Thruster for an Interplanetary CubeSat," *Proceedings of The IEEE,* Vol. 106, No. 3, March 2018. DOI: 10.1109/JPROC.2018.2799898

[2] Dong, K., et al., "Dynamic Influence of Propellant Sloshing Estimation Using Hybrid: Mechanical Analogy and CFD," *Transactions of the Japan Society for Aeronautical and Space Sciences,* Vol. 52, Issue 177, pp 144-151, DOI: 10.2322/tjsass.52.144

[3] Veldman, A.E.P., et al., "The numerical simulation of liquid sloshing on board spacecraft," Journal of Computational Physics Vol. 224, 2007 pp 82-99 DOI: 10.1016/j.jcp.2006.12.020

[4] Mason, P., and Starin, S., "Propellant Slosh Analysis for the Solar Dynamics Observatory," NASA NTRS 20050243594, 2005

[5] Dodge, F. T., *The New "Dynamic Behavior of Liquids in Moving Containers,"* Southwest Research Institute, San Antonio, 2000.

[6] De Souza, L. C. G., and De Souza, A. G., "Satellite Attitude Control System Design Considering the Fuel Slosh Dynamics," *Journal of Shock and Vibration,* published online 26 June 2014. DOI: 10.1155/2014/260206

[7] Koenig A.W., Macintosh B., and D'Amico, S., "Formation Design of Distributed Telescopes in Earth Orbit for Astrophysics Applications," *Journal of Spacecraft and Rockets,* Vol. 56, Issue 5, pp. 1462-1477 DOI: 10.2514/1.A34420

[8] Kamalabadi, F., et al., CubeSat Ideas Lab: Collaborative Research: Virtual Super-resolution Optics with Reconfigurable Swarms (VISORS), National Science Foundation proposal number 1936663, May 2019.

[9] Smith., M. W., et al., "On-Orbit Results and Lessons Learned from the ASTERIA Space Telescope Mission," *32nd AIAA/USU Conference on Small Satellites*, Logan, UT, August 2018

[10] Laesecke, A., and Defibaugh D., "Viscosity of 1,1,1,2,3,3-Hexafluoropropane and 1,1,1,3,3,3-Hexafluoropropane at Saturated-Liquid Conditions from 262 K to 353 K," *Journal of Chemical & Engineering Data,* Vol. 41, pp 59-62. DOI: 10.1021/je950206t

[11] Mulero, A., Cachadina I., and Parra, M. I., "Recommended Correlations for the Surface Tension of Common Fluids," *Journal of Physical and Chemical Reference Data,* published online 19 December 2012. DOI: 10.1063/1.4768782

[12] Coulter, Nolan, "Design of An Attitude Control System for a Spacecraft with Propellant Slosh Dynamics," Masters Thesis, Embry-Riddle Aeronautical University, Daytona Beach, Florida, December 2018.

[13] Franquiz. F. J. et al., "Attitude Determination and Control Systems Design for a 6U CubeSat for Proximity Operations and Rendezvous," *AIAA/AAS Astrodynamics Specialist Conference,* published online 1 August 2014. DOI: 10.2514/6.2014-4421

[14] Ogata, K. *System Dynamics,* 4^{th} ed, Pearson Prentice Hall, New Jersey, 2004